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Macroeconomic Model of Long-term Capital, Short-term Capital and Credit Creation

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Abstract: In recent days, the world economy faces business recession due to problematic credit contraction. In this paper we investigate effects of credit contraction on supply side of the economy. In the economy where uncertainty is prevailing due to the high speed economic change the economic agent devaluates real assets like physical capital compared to nominal assets like money holdings. As a consequence,capital accumulation and supply capacity are diminished and the stagnation is prolonged.
Keywords: Long-term capital, Short-term capital, Credit creation, Credit contraction

1.Introduction

In this paper we investigate effects of credit contraction on supply side of the economy.Long-term capital or physical capital has two positive effects on production.One is the direct effect that arises from the fact that long-term capital is factor of production. The other is the indirect effect that arises from the fact that long-term capital is evaluated as security for a bank loan. By the way, firms need money for smooth production. Bank sector lends this money to the firm under a certain criterion. In the advanced economy which is characterized by terms such as creative destruction or uncertainty, value of long-term capital becomes so instable that the firm cannot obtain desirous amount of a loan. In such a situation, the firm needs short-term capital to borrow the loan from the bank.

In the situation that the bank sector of the economy constricts credit loan to the production sector based on the physical capital, the optimal physical capital stock and thus production level could decline. This type of recession cannot be overcome by government expenditure because the cause of depression is not the shortage of effective demand but the decline of capacity of supply side.

The plan of the paper is as follows. Section 2 deals with the production function and the credit creation function. The production function is expanded to include money. The

perspective that money facilitates transaction is introduced into the production function. Money is supplied by the bank through the credit creation function and the amount of credit creation is depends on two types of mortgage.One is the long-term capital and the other is the short-term capital. In section 3 the simple macroeconomic model with money and credit creation is presented. In section 4 we conclude the paper.

2.Production and Credit Creation

Cobb-Douglas production function is widely used in economic analysis because of its desirable features.

Sinai and Stokes (1972) estimate production functions with and without real money balances and then insists that real money balances are an important input in the production function. They said also that trend term which is understood to represent technological progress in estimating production function may be a proxy for real money balances.

This view of real money balances is criticized by many researches later mainly because of reverse causality. But it is possible to treat money at least as proxy for technological progress or existence to facilitate transaction.

Therefore, we assume production function indicated in equation (1).

$$Y = AK_l^{\alpha}(ML)^{1-\alpha}, \quad 0 \leq \alpha \leq 1. \tag{1}$$

It can be captured by this production function that money facilitates production process.

Money is created through lending activities by the bank sector. In principle, amount of lending for a certain firm depend on sum of discounted future flow of profit the firm will be able to earn. In the real economy, however, amount of a loan a firm can borrow from a bank depends heavily on mortgages.

We can consider that there are two types of mortgages. One is the real asset such as land, factory, machine and equipment. In this paper, we call these real assets long-term capital. The other is the financial asset such as cash, money and liquidity. In this paper, we call these financial assets short-term capital.

Amount of a loan depends on both of long-term capital and short-term capital.

Therefore, we can suppose credit creation function indicated in equation (2).

$$M = K_l^{\gamma}K_s^{1-\gamma}, \quad 0 \leq \gamma \leq 1. \tag{2}$$

It is supposed that γ is in interval [0, 1]. This means that money supply through credit creation depends on both of long-term capital and short-term capital. As an extreme case, consider the situation that $\gamma = 1$. In this case short-term capital is not evaluated as mortgage and the amount of lending is same as the amount of long-term capital. In other words short-term capital is treated as waste. In the economy where growth is mainly due to

physical capital accumulation, investment in long-term capital is worthy and short-term capital is worthless. That is, in the economy like this γ is thought to be near 1.

On the other hand, if $\gamma = 0$, amount of a loan is same as the amount of shortterm capital. In this case long-term capital is not evaluated at all. In the economy where the main engine of growth is research and development as endogenous growth theories imply, keeping short-term capital is very important because it is converted into the long-term capital whenever it is thought to be best timing. For example, consider automobile makers. It is no clear now which of the more efficient internal combustion engine or electric power will becomes the next mainstream of the power source for automobiles. In such a situation, it is very risky for automobile makers to invest all of money holdings to the single candidate. If the firm invest all short-term capital to research and development of new generation combustion engine and it turns out that the mainstream will be electric power, invested capital for internal combustion engine has little value no matter what the invented combustion engine is efficient because social capital will be changed to suit for the next mainstream.

Therefore, if there is so much uncertainty about the future, the value of the firm and the amount of bank credit the firm can borrow depend mainly on the amount of short-term capital the firm holds.

Taken together these points, we can suppose that γ in equation (2) becomes smaller and smaller as uncertainty about the future grows.

By inserting (2) into (1), we have,

$$Y = AK_l^{\alpha+\gamma(1-\alpha)}K_s^{(1-\alpha)(1-\gamma)}L^{1-\alpha}.$$

This resulting production function exhibits increasing returns to scale with respect to physical capital and labor1. This feature is consistent with many empirical researches2.

3.Simple Static Model

3.1 Setup of the Model

In this section, we examine the economic system shown in figure 1 and analyze how its production level is determined through relationship between the household, the firm and the bank. Production function of consumer goods is assumed to be (1) and credit creation function is assumed to be (2).

The household funds the firm by its investment on the long-term capital *Kl* and makes a deposit *Ks* on the bank. This deposit serves as both the primary deposit and one of the

1 Here, $\alpha + \gamma(1-\alpha) + (1-\alpha) = 1 + \gamma(1-\alpha) \geq 1$ as long as $0 \leq \alpha \leq 1$ and $0 \leq \gamma \leq 1$.

2 For example, Sinai and Stokes(1972) and Bodkin and Klein(1967).

mortgage for credit creation *M*. The firm pays dividend *rKl* to the household and pays interest *iM* to the bank for loan. Finally, the bank pays interest *jKs* to the household for his deposit.

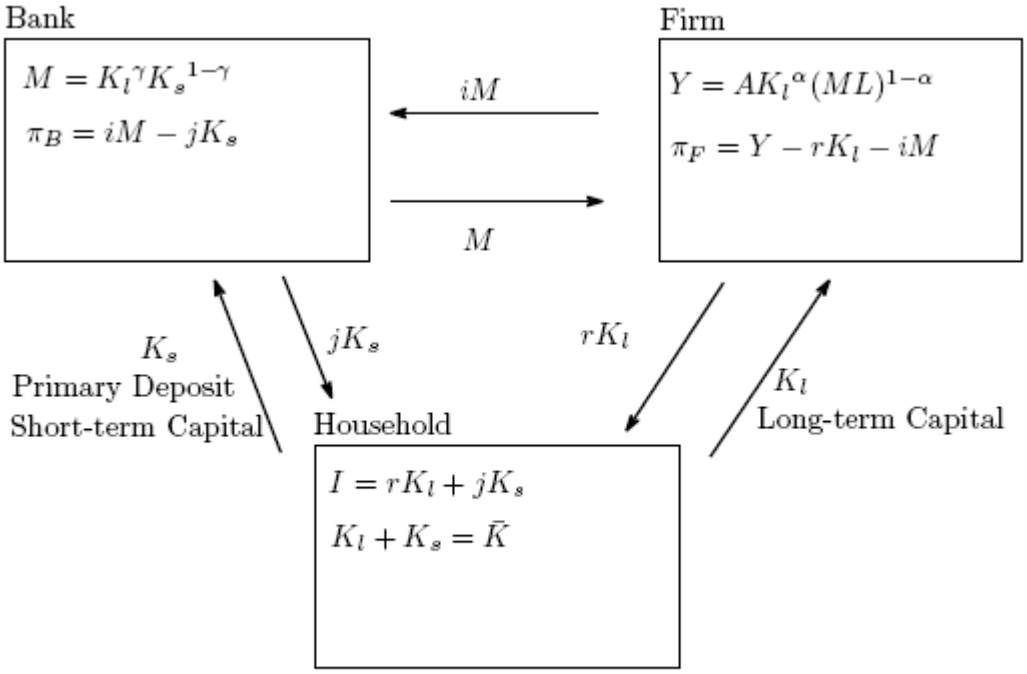

Figure 1

3.2 Decentralized Economy

In this subsection, we analyze decentralized economy.

3.2.1 Behavior of the Household

Household has constant amount of assets which can be used as long-term capital *Kl* or short-term capital *Ks*. Household determines ratio of long-term capital to short-term capital. Long-term capital becomes fund for the firm and thus could be interpreted as investment in stocks. In contrast, short-term capital is deposited into the bank. This means that short-term capital could be interpreted as money or liquidity assets and could be regarded as primary money on deposit.

Assume that total income of household consists of two different incomes. As indicated in figure 1, *rKl* is rental price payment for long-term capital from the firm and *jKs* is interests paid to short-term capital which is saved in the bank. For simplicity, we assume that household supplies labor force to the firm inelastically to the wage and that the wage rate is 031. Therefore, total income *I* is,

$$I = rK_l + jK_s. \tag{3}$$

1 Because the main purpose of the paper is to analyze relationship between long-term capital, shortterm capital and production level, we ignore household's labor supply behavior and labor demand of the firm.

Household determines ratio of long-term capital to short-term capital so as to maximize income *I*. Here, it is assumed that the capital endowment takes constant value $\bar{K}$. That is,

$$K_l + K_s = \bar{K}. \tag{4}$$

Define *t* as representing ratio of long-term capital to the capital endowment. Then,

$$K_l = t\bar{K}. \tag{5}$$

From (4) and (5), necessarily,

$$K_s = (1-t)\bar{K}. \tag{6}$$

Using (5) and (6), income of the household can be rewritten to be,

$$I = rt\bar{K} + j(1-t)\bar{K}. \tag{7}$$

Therefore, maximization problem of the household is represented like below.

$$\max_t \; I = rt\bar{K} + j(1-t)\bar{K}. \tag{8}$$

Under the condition that resulting equilibrium is the interior solution the first order condition is,

$$\frac{dI}{dt} = 0 \;\Leftrightarrow\; r\bar{K} - j\bar{K} = 0 \;\Leftrightarrow\; r = j. \tag{9}$$

Equation (9) means that returns from the long-term capital must be same as returns from the short-term capital in order that *t* is in (0, 1). If $r > j$, household sets *t* to 1. Conversely If $r < j$, household sets *t* to 0. For any $t \in (0, 1)$ to be equilibrium, household must allocates its capital endowment so as to equalize *r* and *j* as a result1.

3.2.2 Behavior of the Firm

From the view point presented in the previous section, production function of the firm is assumed to be Cobb-Douglas type and constant returns to scale with respect to *Kl* and *ML*.

$$Y = AK_l^{\alpha}(ML)^{1-\alpha}. \tag{10}$$

The firm is founded by the long-term capital and thus owned by the household. The firm pay rental rate *rKl* to the household. For production, the firm also demands money *M* and borrows it from the bank. The cost of borrowing a unit of money is the interest rate *i* on the loan. Therefore, total payments to borrowing loan are *iM*.

Therefore, profit of the firm is,

$$\pi_F = AK_l^{\alpha}(ML)^{1-\alpha} - rK_l - iM. \tag{11}$$

Maximization problem of the firm is represented as below.

1 Here, we assume that there is not uncertainty in the economy or that there is uncertainty on longterm capital but household is risk-neutral. Regardless of which assumption we adopt, equation (9) is to hold. We can incorporate uncertainty and risk aversion behavior by adding risk premium to *r*.

$$\max_{K_l,\ M} \ \pi_F = AK_l{}^{\alpha}(ML)^{1-\alpha} - rK_l - iM. \tag{12}$$

First order conditions of profit maximization of the firm are,

$$\frac{\partial \pi_F}{\partial K_l} = 0 \quad \Leftrightarrow \quad A\alpha K_l{}^{\alpha-1}(ML)^{1-\alpha} - r = 0 \tag{13}$$

$$\frac{\partial \pi_F}{\partial M} = 0 \quad \Leftrightarrow \quad A(1-\alpha)K_l{}^{\alpha}(ML)^{-\alpha}L - i = 0. \tag{14}$$

Equation (13) is demand function of the firm for long-term capital and equation (14) is demand function for loan.

3.2.3 Behavior of the Bank

The bank can create money and lends it to the firm. The credit creation function is assumed to be (2) in the previous section.

In this model, although it is assumed that firm and household are different agents, the firm being founded by household through its long-term capital, shortterm capital of household is interpreted as firm's money holdings.

$$M = K_l{}^{\gamma}K_s{}^{1-\gamma}. \tag{15}$$

To create money, the bank demands primary deposit. This need is satisfied by investment of the household for short-term capital. The bank's revenue is interest payments from the firm. Let *i* be interest rate for bank loan and *j* interest rate for primary deposit. Therefore, profit of the bank is to be,

$$\pi_B = iK_l{}^{\gamma}K_s{}^{1-\gamma} - jK_s. \tag{16}$$

Maximization problem of the Bank is represented as below.

$$\max_{K_s} \ \pi_B = iK_l{}^{\gamma}K_s{}^{1-\gamma} - jK_s. \tag{17}$$

The first order condition of profit maximization of the bank is,

$$\frac{d\pi_B}{dK_s} = 0 \quad \Leftrightarrow \quad i(1-\gamma)K_l{}^{\gamma}K_s{}^{-\gamma} - j = 0. \tag{18}$$

Equation (18) is demand function of the bank for short-term capital.

3.2.4 Equilibrium of Decentralized Economy

From (9) and (13), we have,

$$j = A\alpha K_l{}^{\alpha-1}(ML)^{1-\alpha}. \tag{19}$$

By inserting (14) and (19) into (18), we have,

$$A(1-\alpha)K_l{}^{\alpha}(ML)^{-\alpha}L \cdot (1-\gamma)K_l{}^{\gamma}K_s{}^{-\gamma} = A\alpha K_l{}^{\alpha-1}(ML)^{1-\alpha},$$

$$\Leftrightarrow \ (1-\alpha)(1-\gamma)K_l{}^{\gamma}K_s{}^{-\gamma} = \alpha K_l{}^{-1}M. \tag{20}$$

Here, we think of market clearing conditions. In the economy, there are three different market where *Kl*, *Ks* and *M* are traded separately. Market equilibrium condition for *Kl* is,

$$K_l = t\bar{K}. \tag{21}$$

Left hand side of (21) is the demand for long-term capital by the firm and right hand side is supply of it from the household. Just like as the case of *Kl*, other two market clearing conditions are,

$$K_s = (1-t)\bar{K}, \tag{22}$$
$$M = {K_l}^{\gamma}{K_s}^{1-\gamma} = (t\bar{K})^{\gamma}\{(1-t)\bar{K}\}^{1-\gamma}. \tag{23}$$

By inserting (21), (22) and (23) into (20), we have,

$$\begin{aligned} & (1-\alpha)(1-\gamma)(t\bar{K})^{\gamma}\{(1-t)\bar{K}\}^{-\gamma} = \alpha(t\bar{K})^{-1}(t\bar{K})^{\gamma}\{(1-t)\bar{K}\}^{1-\gamma}, \\ \Leftrightarrow\ & (1-\alpha)(1-\gamma) = \alpha(1-t)t^{-1}. \end{aligned} \tag{24}$$

By solving (24) with respect to *t*, we can derive,

$$t = \frac{\alpha}{1-\gamma(1-\alpha)}. \tag{25}$$

From (25), we can calculate long-term and short-term capital in the market equilibrium.

$$K_l = \frac{\alpha}{1-\gamma(1-\alpha)}\bar{K}, \tag{26}$$

$$K_s = \frac{(1-\alpha)(1-\gamma)}{1-\gamma(1-\alpha)}\bar{K}. \tag{27}$$

From (26) and (27), we obtain ratio of long-term capital to short-term capital as below.

$$\frac{K_l}{K_s} = \frac{\alpha}{(1-\alpha)(1-\gamma)}. \tag{28}$$

Production level in the equilibrium can be calculated as follows.

$$\begin{aligned} Y &= AK_l{}^{\alpha}(ML)^{1-\alpha} = AK_l{}^{\alpha}({K_l}^{\gamma}{K_s}^{1-\gamma}L)^{1-\alpha} = AK_l{}^{\alpha}\left\{\left(\frac{K_l}{K_s}\right)^{\gamma}K_sL\right\}^{1-\alpha}, \\ &= AK_l{}^{\alpha}\left(\frac{K_l}{K_s}\right)^{\gamma(1-\alpha)}{K_s}^{1-\alpha}L^{1-\alpha} = A\left(\frac{K_l}{K_s}\right)^{\alpha}\left(\frac{K_l}{K_s}\right)^{\gamma(1-\alpha)}K_sL^{1-\alpha}, \\ &= A\left(\frac{K_l}{K_s}\right)^{\alpha+\gamma(1-\alpha)}K_sL^{1-\alpha}, \\ &= A\left(\frac{\alpha}{(1-\alpha)(1-\gamma)}\right)^{\alpha+\gamma(1-\alpha)}\frac{(1-\alpha)(1-\gamma)}{1-\gamma(1-\alpha)}\bar{K}L^{1-\alpha}. \end{aligned} \tag{29}$$

3.3 Social Planner's Problem

In the previous subsection, we analyzed outcomes of decentralized economy. Equilibrium level of production in the decentralized economy is not optimal because of the existence of external effect that arises from the fact that long-term capital enters into both factors of production; physical capital itself as long-term capital and multiplier for labor as short-term capital. That is, long-term capital contributes to production not only directly as

physical capital but also secondary as a mortgage for bank loan. The firm does not take care of this secondary effect because credit creation is made by bank sector and its evaluating process is not known to the firm explicitly. In other words, bank's behavior about credit loan is black box for the firm. This is why capital allocation is not optimal in decentralized economy.

In this subsection, socially optimal allocation is investigated by solving social planner's problem.

Social planner's aim is to maximize production given that production function is (1), credit creation function is (2) and endowment of total capital is constant. Therefore, maximization problem is set up as below.

$$\begin{aligned} &\max\ Y = AK_l{}^{\alpha}(K_l{}^{\gamma}K_s{}^{1-\gamma}L)^{1-\alpha}, \\ &\text{s.t.}\quad K_l + K_s = \bar{K},\quad \bar{K} \text{ is given and constant.} \end{aligned}$$

Social planner determines *Kl* and *Ks* so as to maximize *Y* under the constraint that total capital endowment is $\bar{K}$. Define t^* as ratio of *Kl* to $\bar{K}$ which is determined by social planner. Then, maximization problem is represented as below.

$$\begin{aligned} \max_{t^*}\ Y &= A(t^*\bar{K})^{\alpha}\left[(t^*\bar{K})^{\gamma}\{(1-t^*)\bar{K}\}\right]^{1-\gamma}L\Big]^{1-\alpha} \\ &= A(t^*\bar{K})^{\alpha+\gamma(1-\alpha)}\{(1-t^*)\bar{K}\}^{(1-\alpha)(1-\gamma)}L^{1-\alpha} \end{aligned} \tag{30}$$

First order condition to maximize *Y* is simply $dY/dt^*=0$, that is,

$$\begin{aligned} &A\{\alpha+\gamma(1-\alpha)\}(t^*\bar{K})^{\alpha+\gamma(1-\alpha)-1}\bar{K}\{(1-t^*)\bar{K}\}^{(1-\alpha)(1-\gamma)}L^{1-\alpha} \\ &-A(1-\alpha)(1-\gamma)(t^*\bar{K})^{\alpha+\gamma(1-\alpha)}\{(1-t^*)\bar{K}\}^{(1-\alpha)(1-\gamma)-1}\bar{K}L^{1-\alpha}=0 \end{aligned} \tag{31}$$

By solving equation (31) respect to t^*, we can derive optimal t^*.

$$t^* = \alpha+\gamma(1-\alpha). \tag{32}$$

From (32), socially optimal long-term capital K^*l and short-term capital K^*s are obtained.

$$K_l^* = \{\alpha+\gamma(1-\alpha)\}\bar{K}, \tag{33}$$

$$K_s^* = (1-\alpha)(1-\gamma)\bar{K}. \tag{34}$$

Optimal ratio of long-term capital to short-term capital is,

$$\frac{K_l^*}{K_s^*} = \frac{\alpha+\gamma(1-\alpha)}{(1-\alpha)(1-\gamma)}. \tag{35}$$

Just like as the case of decentralized economy, we can calculate socially planned production level as below.

$$Y^* = A\left(\frac{\alpha+\gamma(1-\alpha)}{(1-\alpha)(1-\gamma)}\right)^{\alpha+\gamma(1-\alpha)}\{1-\alpha-\gamma(1-\alpha)\}\bar{K}L^{1-\alpha}. \tag{36}$$

3.4 Comparative Statics

3.4.1 Relation between t and γ

In figure 2, DE curve depicts the relationship between γ and t in decentralized economy. Ratio of long-term capital to short-term capital which is chosen by household is an increasing function of γ. This means that when uncertainty about the value of long-term capital fade away gradually the economic agent invests more and more in long-term capital.

On the other hand, SP line indicates the optimal ratio of long-term capital to short-term capital which is set by social planner under the condition that γ is exogenous parameter also for the social planner.

Comparing DE curve with SP line, we see that in decentralized economy t is lower than the optimal $t*$ for any γ. This is because in the decentralized economy agents do not internalize external effects of long-term capital. That is, in decentralized economy there is insufficient long-term capital compared with socially planned economy. Thus, the government of decentralized economy makes it possible to achieve social optimum by subsidizing investment in long-term capital or taxing on
short-term capital.

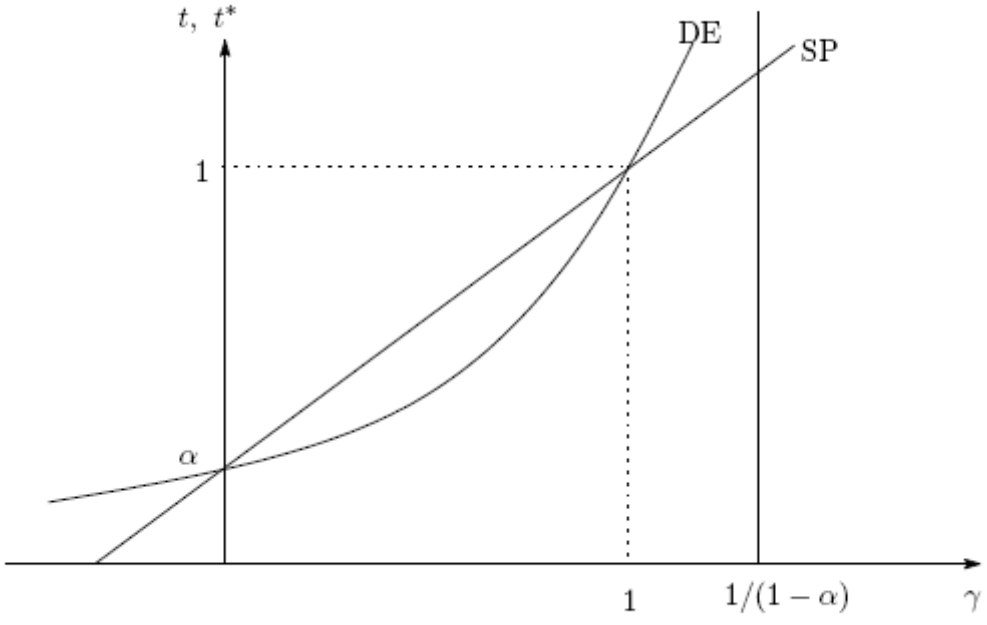

Figure 2

3.4.2 Relation between Y and γ

Relationships between production level Y or $Y*$ and γ for both decentralized economy and social planner are depicted in figure 31. DE curve depicts the relationship between γ and Y in decentralized economy. It is shown that Y is an increasing function of γ.

When γ is relatively small, long-term capital is small compared to short-term capital. As a consequence, production level becomes small. On the other hand, the amount of

1 For simplicity, it is assumed that $\alpha = 1/2$ and $A = \bar{K} = L = 1$ to draw figure 3.

production becomes bigger and bigger when γ closes to 1 and maximized at $\gamma = 1$. This means that if uncertainty about the future comes to disappear all of endowed capital is invested in long-term capital. This feature is same as to the case of social planner.

From the standpoint of the economic policy, the government has some means to increase the production level or GDP. Firstly, the government can avoid the inefficiency of decentralized economy due to the external effect. For any given γ, subsidy to long-term capital or tax on short-term capital rises investment on longterm capital and as a consequence increase production. Secondary, the government can finance short-term capital to the firm or to the household so that they can invest more on long-term capital. Thirdly, the government can get rid of uncertainty to increase value of γ. If we think of relatively small γ as the indication of high level of creative destruction and thus uncertainty, however, we could not regard it good policy to increase γ.

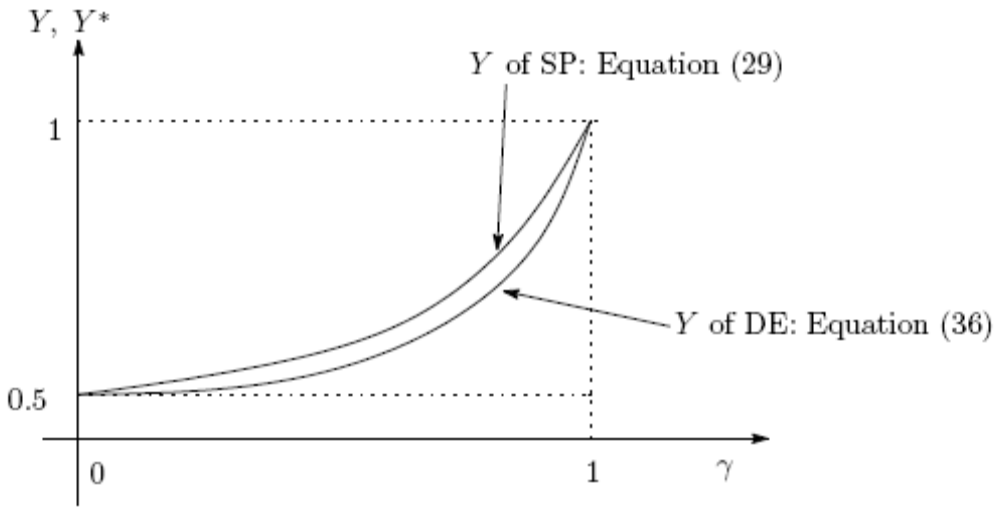

Figure 3

4.Concluding Remarks

We analyzed simple macroeconomic model of credit creation and saw that the cause of declining production is the shortage of physical capital stocks due to the lost value of physical capital as a mortgage and resulting credit contraction. This is a phenomenon of supply side. Therefore, the economic policy the government should implement is not only to stimulate aggregate demand but also to avoid credit contraction so as not to shrink supply capacity. In the long-run supply side is more important than demand side in so far as we rely on competitive markets for resource
allocation.

In this paper the model examined is static. Considering that capital is stock variable accumulated through time, we need a dynamic model. Long-term capital is accumulated by

economic agent's decision to invest. Short-term capital is also accumulated by economic agents. That is, we need dynamic model including two types of capital accumulation.

In this paper it is also assumed that credit creation function is one degree homogeneous and $0 \leq \gamma \leq 1$. As a result, maximum level of production is achieved at $\gamma = 1$. If we consider growth process through creative destruction, however, short-term capital must be stocked to a certain degree. Moreover, we can consider cases where credit creation function is more general as $M = K_l^{\gamma} K_s^{\eta}$. And then, it may be possible to suppose cases where $\gamma > 1$, $\eta > 1$, $\gamma < 0$, $\eta < 0$. Combination of $\gamma > 1$ and $\eta < 0$ may be suitable for the intensive physical capital accumulating economy where catch-up growth is main engine of social progress and the priority of society is to accumulate physical capital. On the other hand, combination of $\gamma < 0$ and $\eta > 1$ may be suitable for the economy that confronts with revolution.

These points are left for further research.

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