

[JP] Hiroshi OHNISHI**Contact Information**

Address: Graduate School of Economics, Kyoto University
Sakyo-ku, Kyoto, 606-8501, Japan

Telephone: 81-75-753-3465 (office)

Fax: 81-75-753-3492

Mobile: 81-905465-8287

Email Address: ohnishi@econ.kyoto-u.ac.jp

Homepage: <http://www.econ.kyoto-u.ac.jp/~chousa/profile/ohnishi.html>

Hiroshi Ohnishi, born in 1956, is a professor of Graduate School of Economics of Kyoto University, vice-dean of Shanghai Center of Kyoto University, vice-chairman of WAPE. He has three research fields. One is a new mathematical modeling of Historical Materialism. The second one is rebuilding statistical epistemology of social statistics, and the last one is quantitative analysis of the East Asian economies, especially China.

He has published more than 200 articles in different journals such as "Political Economy Quarterly", "Statistics", "Journal of Econometric Study of Northeast Asia," and has written or compiled over 40 books. Among them, 10 books are single author books, including one book translated into Russian, two books translated into Chinese and Korean.

Uneven Development of the World Economy: from Krugman to Lenin

[JP] Hiroshi OHNISHI

The collapse of the American economy has revealed the fundamental weight shift of the world economy. In my understanding, the shift is due to the “uneven development” that Lenin defined in *Imperialism*, in which he also proved the validity of this historical law. First, to achieve greater clarity, this paper reviews Krugman’s model of “uneven development” between two countries, namely, South and North. Second, we show that he has misunderstood Lenin’s theory and that Lenin’s theory is more realistic than Krugman’s.

I. Krugman’s “Uneven Development”

1. Krugman’s “Uneven Development”

In Western economics, the North-South divergence of the world economy has been explained by Ricardo’s theory of comparative costs between industrial countries and agricultural countries, or by the Heckscher-Ohlin-Samuelson model with capital-intensive countries and labor-intensive countries, or sometimes by Dutt’s learning-by-doing model with more experienced countries in production and less experienced countries (Dutt (1986)). Besides these, however, we also focus on Krugman (1981) because he refers to Lenin’s model. He has demonstrated that many industrialized countries cumulatively accumulate much more capital than the less-industrialized countries under the assumption of an increasing return on technology. First, we present his model.

Krugman’s model uses two countries, that is, “North,” which is expressed by the suffix “N” and “South,” which is expressed by the suffix “S.” Here, he assumed that both countries have the same amount of labor force, $\bar{L} = L_N = L_S$, and produce two goods—manufacturing good M and an agricultural product A . He also assumed a single world price of manufacturing goods in terms of agricultural products, P_m . In other words, a single world price of agricultural products was set as the unit price.

Manufacturing production was assumed as a function of capital input and labor input, and its technology was assumed to provide increasing returns, that is

$$\begin{aligned} c_N &= c(K_N) & c_S &= c(K_S) & c' &< 0 \\ v_N &= v(K_N) & v_S &= v(K_S) & v' &< 0 \end{aligned}$$

, where c and v denote the unit capital and labor requirements, respectively, resulting in the following relationships:

$$M_N = K_N / c_N \quad M_S = K_S / c_S \quad (1)$$

Agricultural products were assumed to be produced by labor alone, and he chose units such that one unit of labor produces one unit of agricultural goods. An additional assumption of full employment led him to the following agricultural production functions:

$$A_N = \bar{L} - v_N M_N \quad A_S = \bar{L} - v_S M_S$$

In addition, labor forces were assumed to consume agricultural goods alone, and their savings ratios were assumed to be zero, which means that the unit labor cost is one.

Based on the abovementioned equations, the profit rates of the manufacturing sectors were as follows:

$$\rho_N = (P_M M_N - v_N M_N) / K_N = (P_M - v_N) / c_N$$

$$\rho_S = (P_M M_S - v_S M_S) / K_S = (P_M - v_S) / c_S$$

Because c and v are functions of the capital stocks, these equations can be rewritten as follows:

$$\rho_N = \rho(P_M, K_N) \quad \rho_S = \rho(P_M, K_S), \quad (2)$$

where $\frac{\partial \rho}{\partial P_M} > 0$, $\frac{\partial \rho}{\partial K} > 0$, because $c' < 0$ and $\rho_N > \rho_S$ because $K_N = K_S$. Krugman investigated the North-South relationship by using the framework outlined above. His first case includes international trade but excludes international capital movement.

In this case, each country's capital accumulation can be shown as follows:

$$\dot{K}_N / K_N = \rho_N \quad \dot{K}_S / K_S = \rho_S \quad (3)$$

where $\dot{X} = dX / dt$, and because $\rho_N > \rho_S$,

$$\dot{K}_N / K_N > \dot{K}_S / K_S.$$

This means that the capital accumulation in the North is faster than it is in the South, but to represent the dynamics in the K_N - K_S diagram, he continued the analysis. For this purpose, he changed the assumption to that of 100 μ percent of the workers' income being spent on the purchase of manufacturing goods. In this case, the supply-demand balance of manufacturing goods became the following:

$$P_M (M_N + M_S) = \mu (L_N + L_S) = 2\mu \bar{L}.$$

This was then rewritten as

$$P_M = \frac{2\mu \bar{L}}{\left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right)}. \quad (4)$$

Finally, Krugman combined (2), (3), and (4) and introduced the following:

$$\dot{K}_N / K_N = g(K_N, K_S) (= \rho_N) \quad \dot{K}_S / K_S = g(K_N, K_S) (= \rho_S). \quad (5)$$

While these are the basic relations required in order to determine the K_N - K_S dynamics, we

need the division lines, which divide the areas into K_N and K_S and also denote the increasing and decreasing areas. These lines are $\rho_N = 0$ and $\rho_S = 0$, and as given in appendix A, Krugman introduced the following relations:

$$\frac{\partial K_N}{\partial K_S} \Big|_{\rho=0} < 0$$

$$\left| \frac{\partial K_N}{\partial K_S} \Big|_{\rho_S=0} \right| < \left| \frac{\partial K_N}{\partial K_S} \Big|_{\rho_N=0} \right|$$

These show that both lines $\rho_N = 0$ and $\rho_S = 0$ are downward sloping and that line $\rho_N = 0$ is steeper than line $\rho_S = 0$, as shown in figure 1.

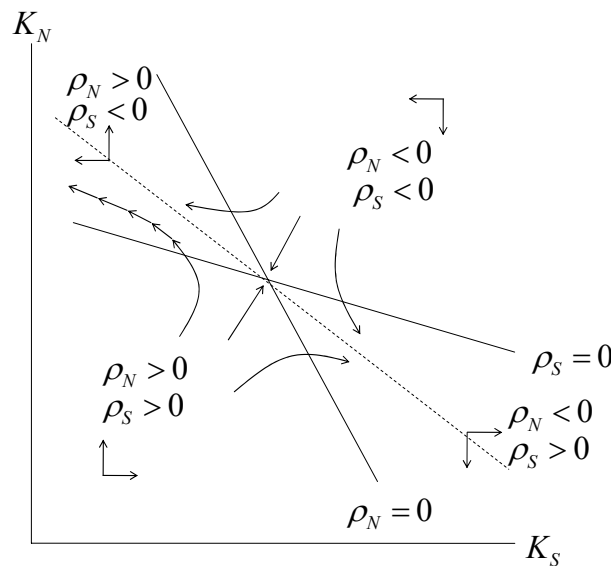


Figure 1

Therefore, the K_N - K_S dynamics take the form shown in figure 1, where the intersection of both lines $\rho_N = 0$ and $\rho_S = 0$ is at a 45 degree angle. It illustrates that if the North-South relation starts from A—where the Northern capital stock is larger than the Southern capital stock—the situation moves in the upper-right direction and then toward B. This implies that the Northern manufacturing industry grows at a faster rate and finally, the South will specialize in agriculture. That is Krugman’s theory that international trade leads to uneven development even if there is no international capital movement.

2. Krugman’s “Lenin Model”

In Western economics, the type of dynamics as proposed by Krugman, which amplifies the North-South differences, is called “divergence,” but the reality of the world economy, at least in East Asia, is opposite. The lowest growth is in Japan, and the highest in China. The world economy is also similar in the sense that the US and European

economies have collapsed and the center of gravity of the world economy is going to shift to the biggest developing country—China. However, Krugman was unable to comprehend this fact.

Furthermore, Krugman also misunderstood Lenin’s theory of *Imperialism*. In his understanding, Lenin and Hobson wanted to illustrate the North-South relations when the capital moves internationally in the second stage of the internationalization of the world economy. He understood this correctly, but mistook the further aspects because he wanted to understand Lenin by using his model, which we shall now examine.

Because Krugman changed his model to a model with perfect capital mobility, represented by the orbits of K_N and K_S attached to the vertical axis in figure 2 and depicted as “A→→→B.” This is because all the capital moves quickly to the countries with higher profit rates under the assumption of perfect capital mobility; then, K_N reaches K_{MAX} , which implies that North specializes in manufacturing goods. In K_{MAX} , which is denoted as B, the reserve army in the agricultural sector disappears (this indicates a turning point in terms of development economics), and wages rise to push out capital to South by pressuring the Northern profit rates to a lower level than in South. Therefore, from here, the world situation starts to move to C from B as shown in figure 2, due to the international capital movement. In other words, first the “A→→→B” process continues, maintaining the specialization in the manufacturing sector promoted by free trade, and then, in the “B→→→C” process, the international capital moves to South. In this way, Krugman’s model has been understood to express two stages of internationalization: the stage of free trade and the “imperialist” stage.

Furthermore, Krugman illustrated Lenin’s theory of “labor aristocracy” in the “B→→→C” process in the sense that Northern wages are higher than the Southern in this process.

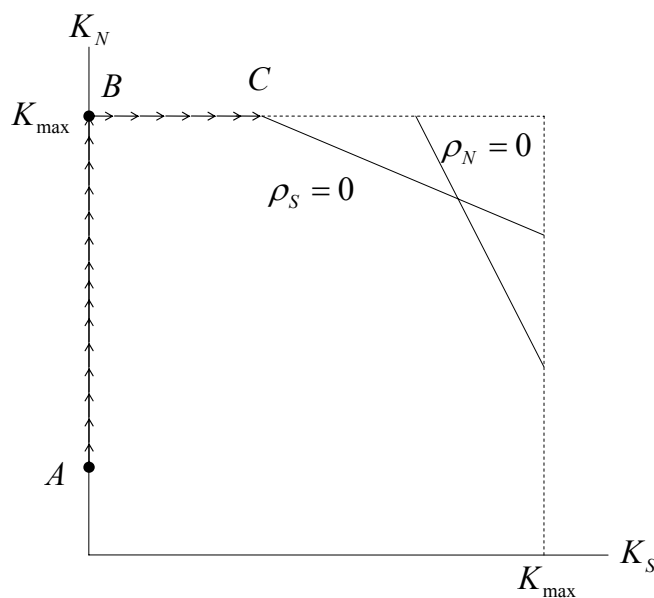


Figure 2

II. Lenin's "Uneven Development"

1. Lenin's "Uneven Development"

As we have seen, some aspects of Lenin's theory were illustrated by Krugman's model. However, I am skeptical that Krugman could understand and express Lenin's basic idea. We must know what Lenin himself stated. The following quotation is from his book *Imperialism*:

"As long as capitalism remains what it is, surplus capital will be utilized not for the purpose of raising the standard of living of the masses in a given country, for this would mean a decline in profits for the capitalists, but for the purpose of increasing profits by exporting capital abroad to the backward countries. In these backward countries profits are usually high, for capital is scarce, the price of land is relatively low, wages are low, raw materials are cheap. The export of capital is made possible by a number of backward countries having already been drawn into world capitalist intercourse; main railways have either been or are being built in those countries, elementary conditions for industrial development have been created, etc. The need to export capital arises from the fact that in a few countries capitalism has become "overripe" and (owing to the backward state of agriculture and the poverty of the masses) capital cannot find a field for profitable investment...."

(Lenin (1917))

Therefore, we can identify two points that are different from Krugman's understanding. They are as follows:

(1) In Krugman's model, South cannot be industrialized after reaching point C in figure 2, but Lenin did not impose such a limitation. Lenin stated that backward capitalism can overtake advanced capitalism and claimed that it also has the right to be redistributed to the colonies. Of course, the advanced capitalism does not agree with this claim, and therefore, this conflict between backward capitalism and advanced capitalism becomes a world war. That is, in the theory of Lenin's imperialist war, backward capitalism's overtaking of advanced capitalism is critical.

(2) Lenin claimed that the reason why backward capitalism's profit rate is low is its scarcity of capital. This means that there is a counter-correlation between profit rates and capital stock, that is, $\frac{\partial \rho}{\partial K} < 0$. Therefore, we must assume the diminishing returns of capital to build Lenin's model. This is diametrically opposite to the framework of Krugman's model.

2. International Trade and Capital Movement under Diminishing Returns of Capital

Therefore, we investigate international trade and capital movement under the diminishing returns of capital assumption.

First, we consider the case with the absence of international capital movement but the presence of trade, as in figure 1. That is, on the lines $\rho_S = 0$ and $\rho_N = 0$, we can introduce

$\frac{\partial K_N}{\partial K_S} \Big|_{\rho=0} < 0$, which means that both of the lines $\rho = 0$ are downward-sloping as in figure 1.

However, the slope of both lines become opposite to that in the Krugman case (see appendix B). That is,

$$\left| \frac{\partial K_N}{\partial K_S} \Big|_{\rho_S=0} \right| > \left| \frac{\partial K_N}{\partial K_S} \Big|_{\rho_N=0} \right|.$$

Therefore, here we can present figure 3 which shows that the two countries' situations tend to be equalized at the intersection of both lines (E). This means that international trade enables backward countries to catch up with advanced capitalism under the condition of diminishing returns of capital.

Second, we consider the next case with international movement; however, because here we assume diminishing returns of capital, c' becomes positive and, thus $\rho_N < \rho_S$ when $K_N > K_S$. Therefore, capital moves from North to South until K_N is equalized to K_S . If the speed of this capital movement is unlimited, the situation of both countries can always be represented by a forty-five degree line as in figure 3. However, if this speed is limited, the dynamic path is faster than in the case without capital movement, but that path is basically the same as in figure 3. In either case, our new assumption of diminishing returns of capital leads the world to convergence; that is a decrease in the disparity between South and North.

Furthermore, in the last part of the path denoted as *, K_N decreases while K_S increases. This indicates the deindustrialization in North, which is also reflective of the current situation, namely, a rapid and radical weight shift of the world economy. Krugman could not explain this phenomenon, but Lenin's model can provide a complete explanation for it.

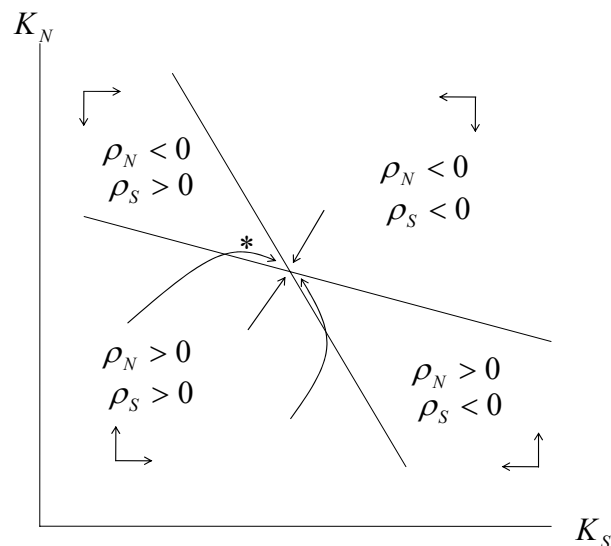


Figure 3

III. Historical Change from the Krugman Case to the Lenin Case

1. From Divergence to Convergence by the Change of Technological Characteristics

However, such a catching-up process of the backward countries was not prevalent even among the Asian Newly Industrializing Economies (NIEs) before the 1970s and among the Southeast Asian countries before 1985. Lenin said “The export of capital is made possible by a number of backward countries having already been drawn into world capitalist intercourse; main railways have either been or are being built in those countries, elementary conditions for industrial development have been created, etc.”

Therefore, the completion of the social infrastructure is a precondition for economic development, and without which backward countries cannot start to catch up. A technological characteristic of such infrastructures is generally an increasing return of capital, and this is assumed by the Krugman model. However, after the completion of such an infrastructure, technological characteristics can cause diminishing returns of capital.

In this sense, here, we could understand that the uneven development of the US, Japan, and other Asian countries in the post-war period must be explained first by the phenomenon of increasing returns of capital and then by that of diminishing returns of capital. Perhaps, every country experiences an increasing return of capital until a certain stage of development, and then, enters the stage of diminishing returns of capital. This kind of technological change is widely known as the “S-shaped production function.” In this sense, such an assumption regarding change of technology is reasonable.¹

¹ Krugman does not always claim divergence. In the second chapter, Krugman(1991a) stated

IV.S-shaped Production Function in Asian-Pacific Countries and the Change of Technology

Furthermore, this kind of technological change can be introduced if we assume that the general path of economic development has three phases: (1) low economic growth due to capital constraint, (2) high economic growth by the dissolution of the constraint, and (3) low economic growth due to labor constraint. Under this assumption, these three phases can be illustrated as the A→B phase, the B→C phase, and the C→D phase, respectively in figure 4. Then, the marginal productivity of capital increases in the first phase, becomes highly stable in the second phase, and decreases in the third phase, if this economy has a stable Cobb-Douglas production function $Y = AK^\alpha L^\beta$ and $\alpha + \beta \leq 1$, $\alpha < 1$ (see figure 5).

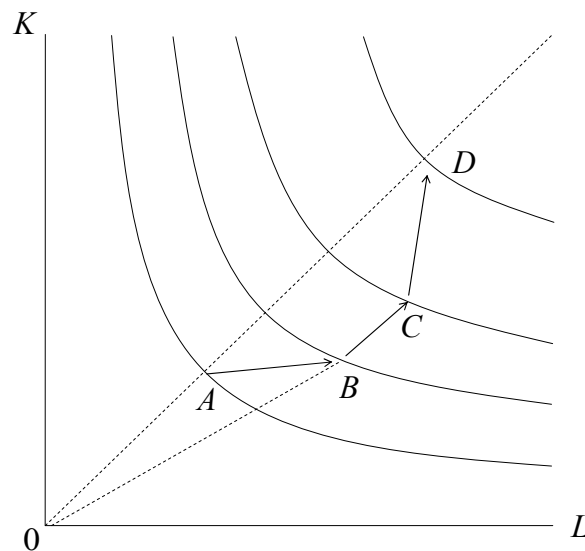


Figure 4

“there is a kind of product cycle, in which emergent new industries initially flourish in localized industrial districts, then disperse as they mature,” on the industrial sector. However, the conditions for such a claim are as follows: too much agglomeration of the labor force, over specialized intermediate goods, and a spillover of the know-how. These do not constitute technological characteristics. Similarly, in the third chapter, a decrease in the transportation cost is shown to constitute the agglomeration cycle of industries, that is. agglomeration → dispersion → agglomeration.

Furthermore, Krugman (1991b) assumed the international free movement of the labor force and studied the effects of transportation costs and the ratio of inter-regionally movable industries. However, this analysis did not have a historical perspective.

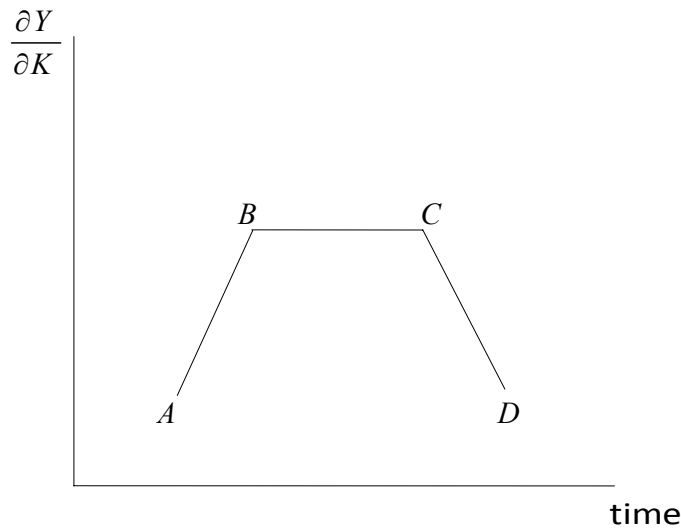


Figure 5

To demonstrate this relationship, first, let us introduce

$$\text{profit rate} = \frac{\partial Y}{\partial K} = \alpha A \left(\frac{K}{L} \right)^{\alpha-1} L^{\alpha+\beta-1} \equiv \alpha A \left(\frac{K}{L} \right)^{\alpha-1} .$$

Thus, the marginal productivity of capital is a decreasing function of the capital-labor ratio (K/L) and can be shown as the slopes of the lines that connect the origin and each point in figure 4.1 Therefore, we can understand the A→B process as constituting a rising phase of the profit rate where the lower speed of capital accumulation than that of labor supply causes the capital-labor ratio to decrease in figure 4.2 and then causes the profit rate to increase. Similarly, we can understand the C→D process constituting a falling phase of the profit rate where the lower speed of the population growth rate than that of capital accumulation causes an increase in the capital-labor ratio, and then causes a decrease in the profit rate. In the B→C process, because the capital-labor ratio is stable, the profit rate is also stable. Such a relationship is exactly the same as that represented by the S-shaped production function, which can be illustrated in a Y-K diagram.³

1 Barro (1991) also noted that a lower capital-labor ratio in developing countries results in a higher marginal productivity of capital, and then leads to a higher growth rate than that of developed countries.

2 An estimation of the marginal productivity of capital in the pre-war period in Japan in chapter seven by Inada, Sekiguchi, and Shoda (1992) was negative in light industries and non-negative in heavy industries. Therefore, a relatively higher weightage was given to the heavy industries than to the light industries might have lead to increasing returns of capital in this period.

3 Strictly speaking, to regard the A→B process as a period of diminishing returns of capital, we should assume K as a function of time (t) and $K''(t) > 0$. In this case, because $c \equiv \frac{K}{AK^\alpha L^\beta} = A^{-1} \left(\frac{K}{L} \right)^\beta K^{1-\alpha-\beta} \equiv A^{-1} \left(\frac{K}{L} \right)^\beta$, c is decreasing as time proceeds $\left(\frac{dc}{dt} < 0 \right)$.

Therefore, $\frac{dc}{dK} \bullet \frac{dK}{dt} \equiv \frac{dc}{dt} < 0$, and because of the above assumption ($K''(t) > 0$), $\frac{dc}{dK} \equiv c'$ must

Therefore, we must note that the characteristics of the technology determine the world system. This idea is surely materialistic and completely consistent with Lenin's theory, which claims that the amount of accumulated capital compared with the labor force (i.e., capital-labor ratio) determines the profit rate and subsequently leads the international capital movement. Furthermore, Lenin's claim that the wage level is lower in backward countries can be understood as a result of the capital-labor ratio by introducing the following equation:

$$\text{wage rate} = \frac{\partial Y}{\partial L} = \beta A \left(\frac{K}{L} \right)^{1-\beta} K^{\alpha+\beta-1} \equiv \beta A \left(\frac{K}{L} \right)^{1-\beta}.$$

This equation indicates that the scarceness of capital results in a lower wage rate. Therefore, Lenin's theory of uneven development can be understood as a theory of return of capital, but more precisely as a theory of the long-term trend of capital-labor ratio.

Then, our last problem is to determine whether there is this type of historical trend of marginal productivity of capital, and in fact, we observe it in the seven Asian-Pacific countries as shown in figures 6 and 7. These two figures are based on our own estimation of capital stock, which is shown in Ohnishi (1998) and Urasaka (1996), and extended. As shown, the Philippines, Thailand, and Indonesia are in the A→B and B→C phase, while USA, Japan, Korea, and Australia are primarily in the C→D phase.¹ In this sense, the abovementioned hypothesis can be regarded as realistic.

V. Conclusion

This paper was the first to review Krugman's understanding of the effect of international trade, capital movement, and Lenin's theory. We could see that Krugman misunderstood Lenin's basic point, and that this misunderstanding came from his technological assumption of increasing returns of capital; therefore, we modified this assumption to one of diminishing returns of capital, and introduced the following implications:

- (1) Lenin's theory of uneven development could be modeled by the assumption of diminishing returns of capital.
- (2) The technologies of the Asian-Pacific countries are moving from a condition of increasing returns of capital to diminishing returns of capital; as a result of this transition, this area has turned toward convergence.
- (3) All of these analyses prove that technology is a key factor that determines the world. This is part of historical materialism.

be negative. This means that there is an increasing return of capital.

¹ Strictly speaking, Japan, Korea, and Australia also demonstrated a short A→B phase and B→C phase in the early stage of the post-war period.

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APPENDIX A The Krugman Case

1)First, we check the sign of the slope of line $\rho_S = 0$. Here, $\rho_S = (P_M - v_S) / c_S = 0$ means that $P_M = v_S$. By combining this equation with equation (4), we obtain the following:

$$2\mu\bar{L} = v_S(K_S) \left\{ \frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right\}. \quad (6)$$

Then, by totally differentiating both sides of this equation, we obtain the following:

$$0 = v_S'(K_S) \left\{ \frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right\} dK_S + v_S(K_S) \left[\frac{c(K_S) - K_S c'(K_S)}{c(K_S)^2} \right] dK_S$$

$$+ v_S(K_S) \left[\frac{c(K_N) - K_N c'(K_N)}{c(K_N)^2} \right] dK_N.$$

Therefore,

$$\frac{dK_N}{dK_S} = - \frac{v_S' \left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right) + v_S \frac{c_S - K_S c_S'}{c_S^2}}{v_S \frac{c_N - K_N c_N'}{c_N^2}}$$

where the denominator and the second term of this numerator are positive because $c' < 0$.

Therefore, if $|v'|$ is not large, $\frac{\partial K_N}{\partial K_S} |_{\rho_S=0} < 0$. This means that the line $\rho_S = 0$ is

downward-sloping.

2) Then, we examine the slope of line $\rho_N = 0$. Exactly as in the former case,

1 Krugman (1991) assumed $\frac{\partial \rho}{\partial K} < \left(\frac{\partial \rho}{\partial P_M} \right) \left(\frac{\partial P_M}{\partial K} \right)$ instead of our assumption $v_N' > v_N \left[\frac{v_N \frac{c_N - K_N c_N'}{c_N^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right]$, $v_S' > v_S \left[\frac{v_S \frac{c_S - K_S c_S'}{c_S^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right]$, and made the lines

$\rho_S = 0$ and $\rho_N = 0$ downward-sloping. These two assumptions are basically the same. The right

side of Krugman's inequality is positive because $\frac{\partial \rho}{\partial P_M} = \frac{1}{c} > 0$,

$$\frac{\partial P_M}{\partial K_N} = \left[\frac{-2\mu \bar{L} \frac{c_N - K_N c_N'}{c_N^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right] < 0, \quad \text{and} \quad \frac{\partial P_M}{\partial K_S} = \left[\frac{-2\mu \bar{L} \frac{c_S - K_S c_S'}{c_S^2}}{\frac{K_N}{c_N} + \frac{K_S}{c_S}} \right] < 0.$$

On the other hand, the left side of Krugman's inequality is $\frac{-v'c - c'(P_M - v)}{c^2}$. Therefore, Krugman's inequality becomes

$$-v'c - c(P_M - v) < c^2 \left\{ \left(\frac{\partial \rho}{\partial P_M} \right) \left(\frac{\partial P_M}{\partial K} \right) \right\}$$

It can be rewritten as

$$v' > - \left\{ c^2 \left(\frac{\partial \rho}{\partial P_M} \right) \left(\frac{\partial P_M}{\partial K} \right) + c'(P_M - v) \right\} / c$$

This is similar to our assumption that $|v'|$ is not so large.

$$0 = v_N'(K_N) \left\{ \frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right\} dK_N + v_N(K_N) \left[\frac{c(K_S) - K_S c'(K_S)}{c(K_S)^2} \right] dK_S + v_N(K_N) \left[\frac{c(K_N) - K_N c'(K_N)}{c(K_N)^2} \right] dK_N .$$

Therefore,

$$\frac{dK_N}{dK_S} = - \frac{v_N \frac{c_N - K_S c_S'}{c_S^2}}{v_N' \left\{ \frac{K_N}{c_N} + \frac{K_S}{c_S} \right\} + v_N \frac{c_N - K_N c_N'}{c_N^2}} .$$

Here, the denominator can be positive if we again, assume that $|v'|$ is not large. In this case, $\frac{\partial K_N}{\partial K_S} |_{\rho_N=0} < 0$, and this means that line $\rho_N = 0$ is also downward-sloping.

3)Then, we compare the slopes of lines $\rho_S = 0$ and $\rho_N = 0$.

$$\begin{aligned} \frac{dK_N}{dK_S} |_{\rho_N=0} - \frac{dK_N}{dK_S} |_{\rho_S=0} &= - \frac{v_N \frac{c_N - K_S c_S'}{c_S^2}}{v_N' Z + v_N \frac{c_N - K_N c_N'}{c_N^2}} + \frac{v_S' Z + v_S \frac{c_S - K_S c_S'}{c_S^2}}{v_S \frac{c_N - K_N c_N'}{c_N^2}} \\ &= \frac{v_S' Z}{v_S \frac{c_N - K_N c_N'}{c_N^2}} + \frac{v_N' Z v_S \frac{c_S - K_S c_S'}{c_S^2}}{\left(v_N' Z + v_N \frac{c_N - K_N c_N'}{c_N^2} \right) \left(v_S \frac{c_N - K_N c_N'}{c_N^2} \right)} , \end{aligned} \tag{7}$$

where $Z = \frac{K_N}{c_N} + \frac{K_S}{c_S}$, and in this case, the first term of the right side of the above equation is negative because its denominator is positive and its numerator is negative. The second term is also negative because the denominator is positive and the numerator is negative due to the negative sign of v_N' . Therefore, the right side of the above equation is negative, and it results in the slope of line $\rho_N = 0$ being steeper than that of line $\rho_S = 0$. Figure 1 illustrates this relationship.

4)Then, we investigate which side of line $\rho_S = 0$ is $\rho_S > 0$ or $\rho_S < 0$. For this purpose, first, we assume that $\rho_S > 0$ as follows. Here,

$$P_M > v_S,$$

because $\rho_S = \frac{P_M - v_S}{c_S} > 0$. By combining the above inequality with equation (4), we

have the following:

$$2\mu\bar{L} > v_S(K_S) \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right). \quad (8)$$

Here, because $2\mu\bar{L}$ is constant, our problem regarding which side of line $\rho_S = 0$ is $\rho_S > 0$ changes into that of which side of line $\rho_S = 0$ causes the right side of inequality (8) to become smaller than the left. To solve this problem, we assume again that $|v'|$ is not large and differentiate the right side of inequality (7), and we have the following:

$$\frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} = v_S' \left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right) + v_S \frac{c_S - K_S c_S'}{c_S^2} > 0$$

Therefore,

inequality (8) and $\rho_S > 0$ are satisfied in the area under line $\rho_S = 0$, and vice versa.

5) Similar to the previous case, we investigate which side of line $\rho_N = 0$ is $\rho_N > 0$ or $\rho_N < 0$. For this purpose, first we assume $\rho_N > 0$ and then we introduce the following:

$$2\mu\bar{L} > v_N(K_N) \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right). \quad (9)$$

By assuming again that $|v'|$ is not large and differentiating the right side of the above inequality, we have the following:

$$\frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} = v_N' \left(\frac{K_N}{c_N} + \frac{K_S}{c_S} \right) + v_N \frac{c_N - K_N c_N'}{c_N^2} > 0$$

$$\frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} = v_N \frac{c_S - K_S c_S'}{c_S^2} > 0.$$

Therefore, also in this case, inequality (9) and $\rho_N > 0$ are satisfied in the area under line $\rho_N = 0$, and vice versa.

APPENDIX B The Lenin Case

1) With respect to the slope of line $\rho_S = 0$, we replace the former assumption of increasing returns of capital ($v' < 0$, $c' < 0$) with the new assumption of diminishing returns

of capital ($v' > 0$, $c' > 0$), and then, we have $\frac{\partial K_N}{\partial K_S}|_{\rho_S=0} < 0$ without any additional assumptions. This means that line $\rho_S = 0$ is downward-sloping.

2)As in the above case, line $\rho_N = 0$ becomes downward-sloping.

3)Then, we compare the slopes of lines $\rho_S = 0$ and $\rho_N = 0$ in this case. The first term of the right side of equation (7) is positive because both its numerator and denominator are positive, and the second term is also positive, because both the numerator and denominator are positive. Therefore, in this case, line $\rho_S = 0$ is steeper than line $\rho_N = 0$ in contrast to the former case. This relationship is illustrated in figure 3.

4)In this case, with any signs of c' and v' , we have the following:

$$\frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} > 0, \quad \frac{\partial \left[v_S \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} > 0$$

$$\frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_N} > 0, \quad \frac{\partial \left[v_N \left(\frac{K_N}{c(K_N)} + \frac{K_S}{c(K_S)} \right) \right]}{\partial K_S} > 0.$$

Therefore, under line $\rho_N = 0$, $\rho_N > 0$, and under line $\rho_S = 0$, $\rho_S > 0$, and vice versa.