

(http://www.globallearningnj.org/global_ata/principles_of_a_sustainable_society.htm)
Principles of a Sustainable Society, Caring for the Earth: Our Common Future (Our Common Future is a report from the UN World Commission on Environment and Development (WCED) and was published in 1987; retrieved from Wikipedia).
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[JP] Yuuho Yamashita

Contact Information:

Address: Dokkyo University 1-1,
 Gakuen-cho, Soka-shi, Saitama
 340-0042, Japan

Email Address: yamashita76@dokkyo.ac.jp

Yuuho Yamashita, lecturer of Department of Economics, Dokkyo University, got Master's degree in Economics in Kyoto University, March 2000. His research areas include Endogenous Growth Theory, Macroeconomics, and Marxian Economics.

His recent published papers are as follows:

- (1) "A Marxist = Neo-classical Modeling of Capitalism as An Optimal Roundabout Production System", with Hiroshi Ohnishi, *Kyoto University Working Paper*, July 2005
- (2) "Roemer's Exploitation in the Neo-classical 'Marxist Model' of Growth", *Kikan Keizai Riron (Political Economy Quarterly)*, Vol.42, No.3, October, 2005, in Japanese
- (3) "Reconstructing Marxism as a Neoclassical Optimal Growth Model", *Seikei Kenkyu*, Institute of Political Economy, No.78, 2002, with Hiroshi Ohnishi, in Japanese
- (4) "On the Labor as the Primary Factor of Production in 'Marxist Model'", *Keizai Ronso*, Kyoto University, Vol.172, No.3, 2003, with Hiroshi Ohnishi, in Japanese
- (5) "Reconstructing Marxism as a Neoclassical Optimal Growth Model" *Economics Study of Shanghai School*, Vol.11, 2004, with Hiroshi Ohnishi and Roxangul Wufuer, in Chinese

Some Extensions of the Marxist=Neo-classical Growth Model¹

[JP] Yuuho Yamashita ²

Abstract: If the rich agent internalizes the effect of his own labor supply on wage and rental rate, the poor agent does not supply labor force to the firm. It is not efficient for the poor to supply labor to the firm under the situation that he or she has same productivity about capital production as the rich who internalize prices. As a consequence, the interrelation between the poor and the rich disappear. The firm that is funded by the rich employs only the rich. In other words, the firm can be deemed to be self-production by the rich. All consumer goods are produced only by self-production. If the rich agent has higher productivity than the poor with respect to capital production, socially required capital is accumulated by the rich alone. In both cases, individual accumulating path is determined endogenously.

1 The author would like to thank Hiroshi Ohnishi and Hideki Fuziyama for their helpful comments. Needless to say, the author is responsible for any remaining errors.

2 Dokkyo University, yamashita76@dokkyo.ac.jp

1 Introduction

Yamashita(2007) investigates the dynamics of asset differential between the rich and the poor agent under the condition that there exists competitive labor market and concludes that the accumulating path of total amount of capital in the society is determined uniquely but individual accumulating paths are not determined endogenously. So, we cannot forecast whether asset differential will grow or disappear by economic fundamentals. This is because how much amount of capital the economic agent privately owns does not affect his or her total life time utility. In equilibrium, there is no difference between the utility that stems from labor supply and the one that stems from capital accumulation.

This conclusion depends crucially on the assumption that the only difference between the rich and the poor is their initial endowment of capital. The rich has more capital than the poor at initial period.

In this paper, on the other hand, we investigate two situations where the assumption mentioned above is changed.

For the first extension, in the section 2, we assume that the rich agent takes into account the effect of his labor supply on the wage rate and the rental rate. In Yamashita(2007), it is assumed that the only difference between the rich and the poor is the initial endowment of capital and both agent behave as price taker. Here, we assume that the rich agent internalize wage and rental rate. That is to say, the rich behave monopolistically.

For the second extension, in section 3, we investigate the situation where the rich agent is much more productive at the production of the capital than the poor agent. That is, the rich has absolute advantage at capital production and its accumulation.

2 Asymmetric behavior in the Labor Market

2.1 Settings of the Model

In the economy, there are rich agent and poor agent. The rich has much more capital at initial period than the poor. Both types of agent have same technology in production of consumer goods and capital goods. We assume the roundabout production system as below.

$$Y_t = AK_t^\alpha L_1^{1-\alpha} \quad (1)$$

$$K_{t+1} = K_t + BL_2 \quad (2)$$

$$L_1 + L_2 = 1 \quad (3)$$

Equation (1) represents a production function of consumer goods and equation (2)

represents a production function of capital goods¹. Equation (3) is a resource constraint of labor force. Here, individual endowment of labor force is normalized to 1. In this roundabout production system, labor force is substantial factor of production.

The only difference between the rich and the poor is the endowment of capital stock at time 0. It is assumed that

$$K_{R0} > K_{P0}, \quad (4)$$

where K_{R0} is initial endowment of capital for the rich and K_{P0} for the poor. In the economy, there is the firm which is capitalized by the rich. The firm produces consumer goods so as to maximize a profit. The firm employs labor forces of the rich and the poor to produce consumer goods and then pays wage w_t to them at same wage rate. Finally, the firm pays rental price r_t to the rich.

2.2 Behavior of the Poor

The poor allocates his labor force endowment which is normalized to one to three different activities. The first, u_t , is used for the self production of consumer goods.¹ The second, s_t , is used for the supply of labor force to the firm. The third, $1-u_t-s_t$, is used for production of capital goods for his own accumulation.

The poor chooses u_t and s_t so as to maximize his life time utility. The poor's consumption at time t , C_{Pt} , is consisted of self production and wage. That is,

$$C_{Pt} = AK_{Pt}^\alpha (u_t)^{1-\alpha} + w_t s_t. \quad (5)$$

¹ For simplicity, the rate of depreciation is assumed to be zero.

In addition, Capital stock of the poor grows according to equation (6).

$$K_{P_{t+1}} = K_{P_t} + B(1 - u_t - s_t). \quad (6)$$

It is assumed that life time utility of the poor is

$$U = \sum_{t=0}^{\infty} \beta^t C_{P_t} \quad (7)$$

Therefore, Lagrange function of the poor is

$$L_P = \sum_{t=0}^{\infty} \beta^t [AK_{P_t}^{\alpha}(u_t)^{1-\alpha} + w_t s_t + \lambda_t \{K_{P_{t+1}} - K_{P_t} - B(1 - u_t - s_t)\}]. \quad (8)$$

Therefore, the first order conditions are

$$\frac{\partial L}{\partial u_t} = 0 \Leftrightarrow (1 - \alpha)AK_{P_t}^{\alpha}u_t^{-\alpha} + B\lambda_t = 0, \quad (9)$$

$$\frac{\partial L}{\partial s_t} = 0 \Leftrightarrow w_t + B\lambda_t = 0, \quad (10)$$

$$\frac{\partial L}{\partial K_{P_t}} = 0 \Leftrightarrow \alpha AK_{P_t}^{\alpha-1}u_t^{1-\alpha} - \lambda_t + \beta^{-1}\lambda_{t-1} = 0. \quad (11)$$

2.3 Behavior of the Firm

A profit of the firm π is

$$\pi = AK_{R_t}^{\alpha}(s_t + v_t)^{1-\alpha} - r_t K_{R_t} - w_t(s_t + v_t). \quad (12)$$

The first order conditions for profit maximization are

$$(1 - \alpha)AK_{R_t}^{\alpha}(s_t + v_t)^{-\alpha} - w_t = 0, \quad (13)$$

$$\alpha AK_{R_t}^{\alpha-1}(s_t + v_t)^{1-\alpha} - r_t = 0. \quad (14)$$

2.4 Behavior of the Rich

The rich agent invests his capital to the firm and earns rental rate r_t as capitalist. Then, he allocates his labor force into two activities. One, v_t , is allocated the supply of his labor force to the firm. The other, $1 - v_t$, is allocated for production of his own capital. Therefore, the consumption of the rich at time t , C_{R_t} , is

$$C_{Rt} = r_t K_{Rt} + w_t v_t.$$

Finally, capital stock of the rich grows according to equation (15).

$$K_{Rt+1} = K_{Rt} + B(1 - v_t) \quad (15)$$

Here, it is assumed that utility function of the rich is same as the poor.

Therefore, Lagrange function of the rich is

$$L_R = \sum_{t=0}^{\infty} \beta^t [r_t K_{Rt} + w_t v_t + \gamma_t \{K_{Rt+1} - K_{Rt} - B(1 - v_t)\}]. \quad (16)$$

The rich internalizes the effect of labor supply on the wage rate and the effect of capital accumulation on the rental rate. That is, by inserting (13) and (14) into (16), Lagrange function of the rich is modified as follows.

$$L_R = \sum_{t=0}^{\infty} \beta^t \left[\alpha A K_{Rt}^{\alpha} (s_t + v_t)^{1-\alpha} + (1 - \alpha) A K_{Rt}^{\alpha} (s_t + v_t)^{-\alpha} v_t + \gamma_t \{K_{Rt+1} - K_{Rt} - B(1 - v_t)\} \right]. \quad (17)$$

Therefore, the first order conditions are

$$\frac{\partial L}{\partial v_t} = 0 \Leftrightarrow (1 - \alpha) \alpha A K_{Rt}^{\alpha} (s_t + v_t)^{-\alpha} - \alpha(1 - \alpha) A K_{Rt}^{\alpha} (s_t + v_t)^{-\alpha-1} v_t + (1 - \alpha) A K_{Rt}^{\alpha} (s_t + v_t)^{-\alpha} + B \gamma_t = 0 \quad (18)$$

$$\frac{\partial L}{\partial K_{Rt}} = 0 \Leftrightarrow \alpha^2 A K_{Rt}^{\alpha-1} (s_t + v_t)^{1-\alpha} + \alpha(1 - \alpha) A K_{Rt}^{\alpha-1} (s_t + v_t)^{-\alpha} v_t - \gamma_t + \beta^{-1} \gamma_{t-1} = 0. \quad (19)$$

2.5 Dynamics of Capital-Labor Ratio

The dynamics of the economy is represented by nine equations, (6), (9), (10), (11), (13), (14), (15), (18), (19) and initial conditions K_{P0}, K_{R0} . This simultaneous equations system consists of nine variables, $K_{Pt}, K_{Rt}, u_t, s_t, v_t, w_t, r_t, \lambda_t$ and γ_t .

From (9), (10), (13), we can derive

$$K_{Pt}^{\alpha} u_t^{-\alpha} = K_{Rt}^{\alpha} (s_t + v_t)^{-\alpha} \Leftrightarrow \left(\frac{K_{Pt}}{u_t} \right)^{\alpha} = \left(\frac{K_{Rt}}{s_t + v_t} \right)^{\alpha}. \quad (20)$$

Here, we define that

$$k_{Pt} \equiv \frac{K_{Pt}}{u_t}, \quad k_{Rt} \equiv \frac{K_{Rt}}{s_t + v_t}. \quad (21)$$

k_{Pt} and k_{Rt} are the capital-labor ratio in production of consumer goods at time t by the poor and by the firm, respectively. From (20) and (21), we can derive

$$k_{Pt} = k_{Rt}. \quad (22)$$

Equation (22) means that capital-labor ratios between two production fields are equalized for all t . This condition means that total amount of production of consumer goods in the whole society for all t is maximized.

From (9) and (11), we can derive

$$\alpha\beta B k_{Pt}^{\alpha-1} + (1 - \alpha)\beta k_{Pt}^\alpha - (1 - \alpha)k_{Pt-1}^\alpha = 0. \quad (23)$$

Capital-labor ratio of the poor's self production of consumer goods is subject to this difference equation. As is the case for the poor agent, the difference equation regarding to capital-labor ratio of the firm's production can be derived. For the first step, (18) and (19) can be represented by k_{Rt} as follows, respectively.

$$\alpha(1 - \alpha)A k_{Rt}^\alpha - \alpha(1 - \alpha)A k_{Rt}^\alpha \frac{v_t}{s_t + v_t} + (1 - \alpha)A k_{Rt}^\alpha + B\gamma_t = 0 \quad (24)$$

$$\alpha^2 A k_{Rt}^{\alpha-1} + \alpha(1 - \alpha)A k_{Rt}^{\alpha-1} \frac{v_t}{s_t + v_t} - \gamma_t + \beta^{-1}\gamma_{t-1} = 0 \quad (25)$$

From (24), we have

$$\gamma_t = -\frac{(1 + \alpha)(1 - \alpha)A}{B} k_{Rt}^\alpha + \frac{\alpha(1 - \alpha)A}{B} k_{Rt}^\alpha \frac{v_t}{s_t + v_t} \quad (26)$$

Finally, from (25) and (26), we have

$$\alpha^2 \beta B k_{Rt}^{\alpha-1} + \alpha(1 - \alpha)\beta B k_{Rt}^{\alpha-1} \frac{v_t}{s_t + v_t} + (1 - \alpha^2)\beta k_{Rt}^\alpha - \alpha(1 - \alpha)\beta k_{Rt}^\alpha \frac{v_t}{s_t + v_t} - (1 - \alpha^2)k_{Rt-1}^\alpha + \alpha(1 - \alpha)k_{Rt-1}^\alpha \frac{v_{t-1}}{s_{t-1} + v_{t-1}} = 0$$

\Leftrightarrow

$$\left\{ \alpha^2 \beta B + \alpha(1 - \alpha)\beta B \frac{v_t}{s_t + v_t} \right\} k_{Rt}^{\alpha-1} + \left\{ (1 - \alpha^2)\beta - \alpha(1 - \alpha)\beta \frac{v_t}{s_t + v_t} \right\} k_{Rt}^\alpha - \left\{ (1 - \alpha^2) - \alpha(1 - \alpha) \frac{v_{t-1}}{s_{t-1} + v_{t-1}} \right\} k_{Rt-1}^\alpha = 0 \quad (27)$$

Difference equation (27) governs dynamics of capital-labor ratio of firm's production of consumer goods.

2.6 Long-run Equilibrium

In the steady state, capital-labor ratio of poor's self production become constant, and let k^*P represent this constant value. Because k^*P satisfies equation (23),

$$\alpha\beta B k_P^{*\alpha-1} + (1-\alpha)\beta k_P^{*\alpha} - (1-\alpha)k_P^{*\alpha} = 0. \quad (28)$$

Therefore,

$$k_P^* = \frac{\alpha\beta B}{(1-\alpha)(1-\beta)} \quad (29)$$

In a similar way, we can calculate steady state value of capital-labor ratio of the firm k_R^* , given the fact that in the steady state $v_t = 1$ and s_t takes a constant value s^* . Moreover, k_R^* satisfies equation (27). That is to say, in the steady state, following equation must be hold.

$$\left\{ \alpha^2\beta B + \alpha(1-\alpha)\beta B \frac{1}{s^*+1} \right\} k_R^{*\alpha-1} + \left\{ (1-\alpha^2)\beta - \alpha(1-\alpha)\beta \frac{1}{s^*+1} \right\} k_R^{*\alpha} - \left\{ (1-\alpha^2) - \alpha(1-\alpha) \frac{1}{s^*+1} \right\} k_R^{*\alpha} = 0 \quad (30)$$

By solving equation (30) respect to k_R^* ,

$$k_R^* = \frac{\alpha^2\beta B + \alpha(1-\alpha)\beta B \frac{1}{s^*+1}}{(1-\alpha^2)(1-\beta) - \alpha(1-\alpha)(1-\beta) \frac{1}{s^*+1}}. \quad (31)$$

By the way, from (22), $k_{Pt} = k_{Rt}$ for all t . This relationship is also applicable to steady state, i.e. $k_P = k_R^*$. That is,

$$\frac{\alpha\beta B}{(1-\alpha)(1-\beta)} = \frac{\alpha^2\beta B + \alpha(1-\alpha)\beta B \frac{1}{s^*+1}}{(1-\alpha^2)(1-\beta) - \alpha(1-\alpha)(1-\beta) \frac{1}{s^*+1}}. \quad (32)$$

Equation (32) means that

$$s^* = 0. \quad (33)$$

(33) means that at least in the steady state the poor agent does not supply his labor force to the firm.

2.7 Transitional Dynamics

Movement of k_{Pt} is governed by (23). Actually, k_{Pt} does not change over time. In other

words, k_{Pt} take the value of k_P^* not only in the long-run steady state but also in a transitional path to the steady state. Intuitively speaking, this is because we assume the linear felicity function in (7). Linear felicity function means that utility maximization is same as maximizing the amount of production. To maximize the amount of production, it is necessary to keep the most efficient capital-labor ratio over time. Therefore, we have

$$k_{Pt} = k_P^* = \frac{\alpha\beta B}{(1-\alpha)(1-\beta)}, \text{ for all } t. \quad (34)$$

From (22) and (34),

$$k_{Rt} = k_R^* = \frac{\alpha\beta B}{(1-\alpha)(1-\beta)}, \text{ for all } t. \quad (35)$$

From (27) and (35),

$$\frac{\alpha\beta B}{(1-\alpha)(1-\beta)} = \frac{\alpha^2\beta B + \alpha(1-\alpha)\beta B \frac{v_t}{s_t+v_t}}{(1-\alpha^2)(1-\beta) - \alpha(1-\alpha)(1-\beta) \frac{v_t}{s_t+v_t}}. \quad (36)$$

Solving equation (36) with respect to s_t , we have

$$s_t = 0 \text{ for all } t. \quad (37)$$

Therefore, not only in the steady state but also in transitional periods, the poor agent supplies no labor force to the firm. In the economy, there is no interrelation between the poor and the rich and the firm does not employ the poor. Each economic agent in this economy behaves as if he or she were alive only by oneself.

3 Asymmetric Productivity of Capital Goods

So far, it is assumed that there is no difference between the rich and the poor about their technology of production. The only difference is the initial inequality of capital ownership and this inequality is assumed exogenously. From historical point of view, however, it is necessary to explain why the initial inequality had arisen. One possible explanation for this question is that the productivity of capital of the rich is higher than that of the poor. That is to say, consider following production system of capital goods.

$$K_{Pt+1} = K_{Pt} + B_P(1 - u_t - s_t) \quad (38)$$

$$K_{Rt+1} = K_{Rt} + B_R(1 - v_t), \quad (39)$$

here, $B_P < B_R$.

In this case, the conclusion that individual accumulating path is indeterminate changes drastically as follows.

If the socially required amount of capital can be produced only by the rich, the rich produces that amount of capital and supply remaining labor force to the firm. On the other

hand, the poor does not produce any capital and allocate all labor force between self-production of consumer goods and labor supply to the firm so as to equalize capital-labor ratio between two production fields. That is, individual accumulating path is determined endogenously and asset differential between the rich and poor expands. In other words, the poor agent has comparative advantage in producing consumer goods at the firm.

4 Concluding Remarks

In this paper, we investigate two extensions of Yamashita(2007). The main conclusions of this paper are summarized as follows.

If the rich behave monopolistically in the labor market, the poor does not supply labor force to the firm. That is, there is no interrelation between the poor and the rich or the firm. All economic agents behave as if they lived in a desert island and accumulate their own capital so as to maximize their life-time utilities. As a consequence, individual accumulating paths are determined endogenously. Even if the rich want to exploit the poor by internalizing wage and rental rate, the rich cannot extract any surplus. In this sense we cannot formalize exploitation by monopolistic labor market.

If the rich has higher productivity at capital production, the poor does not accumulate capital privately beyond the amount that he or she owns at time 0. It is efficient for the poor to concentrate on production of consumer goods by selfproduction or labor supply to the firm.

In both extensions, indeterminacy of individual accumulating path is eliminated and the unique accumulating path is determined.

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[GR] Alexandros Dagkas

Contact Information:

Email Address: ADAGKAS@NURED.AUTH.GR

DAGKAS, ALEXANDROS (PAPER WITH TSOUKALA)

Dr Alexandros Dagkas is senior lecturer in social history at the Aristotle University of Thessaloniki, Greece. His areas of research are the social movements, the working class movement, and the national question. His work has been honoured by the Academy of Athens.

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