

**[JP] Ryo Kanae**

**Contact Information:**

Email Address: kanaeryo@yahoo.co.jp

Ryo Kanae is a student of the graduate school of Kyoto University.

# Value and Price in the Growth Theory

[JP] Ryo Kanae

## Introduction

Marxian two sector model is treated in Uzawa(1961[2]), Yamashita-Ohnishi(2005 [3]) and Yamashita(2006 [4]), using Neoclassical production functions. These models show that Marxian economics may be reinterpreted in terms of the optimal growth theory. But they don't treat Marxian value theory. In this article, we treat it in the framework of growth theory.

### 1. The basic model

We assumed that society has two sectors of production—the consumption goods sector (the first sector) and the capital goods sector (the second sector). Factors of production are capital goods and labor. Consumption goods are produced by capital goods and labor, and capital goods is only labor. Consumption goods is ultimately only by labor (The Labor theory of value).

We denote by  $Y$ ,  $K$ ,  $L$ , respectively, consumption goods, capital goods, labor. In addition, the allocation of labor  $L$  is divided into these two sectors in the ratio  $L_0 : L_1$ . We assume that the production function for the first sector is of Cobb-Douglas type.

$$Y = AK^\alpha(L_0)^{1-\alpha} \equiv F(K, L_0) \quad (1)$$

Furthermore, we assume that the production function for the second sector is simply as follows.

$$\dot{K} = BL_1 - \delta K \equiv G(L_1) - \delta K \quad (2)$$

$\delta$  signifies the constant depreciation rate.

$\dot{K}$  denotes a differential with respect to time.

We set

$p_0$ : the price of consumption goods

$p_1$ : the price of capital goods

$w$ :nominal wage rate

$R$ :rental rate of capital goods

$r$ :interest rate.

We suppose that capitalists only invest,laborers only consume.That is,

$$p_0Y = wL \quad (3)$$

$$p_1(\dot{K} + \delta K) = RK \quad (4)$$

Since  $\dot{K}=0$  in the steady state,(4) is as follows.

$$p_1\delta = R \quad (5)$$

Following equations hold under these setting.

$$p_0Y_K = R \quad (6)$$

$$p_0Y_L = w \quad (7)$$

$$p_1G_L = w \quad (8)$$

(Here, $Y_K, G_K, G_L$  signify partial differential)

From (7)(8),

$$\frac{p_0}{p_1} = \frac{G_L}{Y_L} \quad (9)$$

holds.

From (5)(6),

$$\frac{p_0}{p_1} = \frac{\delta}{Y_K} \quad (10)$$

holds.

From (9)(10),

$$\frac{1}{\delta}Y_KG_L = Y_L \quad (11)$$

holds.

We may interpret (11) as follows. The right-hand side implies labor productivity of consumption goods.Although  $Y_KG_L$  in the left-hand side implies it, $Y_KG_L$  signifies the fact that capital K produce consumption goods Y after laborers L produced capital K.That is, $Y_KG_L$  signifies roundaboutness of production of consumption goods Y by capital K.Hence, $Y_KG_L$  divided by  $\delta$  equals labor productivity of consumption goods.

From (11),

$$K = \frac{B\alpha}{\delta(1-\alpha)}L_0 \quad (12)$$

holds.

Since  $\dot{K} = 0$  in the steady state, from (2)

$$BL_1 = \delta K \quad (13)$$

From (12)(13),

$$\frac{K}{L} = \frac{B\alpha}{\delta}L \quad (14)$$

In what follows, we shall induce the reproduction schema of the basic model.

In what follows, we shall induce the reproduction schema of the basic model.

First, we consider the price system. In the capital goods sector, fixed capital cost is 0 and wage is  $wL_1$  and product price is  $p_1\delta K$ . In the consumption goods sector, fixed capital cost is  $p_1\delta K$  and wage is  $wL_0$  and product price is  $p_0Y$ .

Secondly, we consider the value system. We let the value of a unit of consumption goods  $Y$ , capital goods  $K$  be  $t_0, t_1$ , respectively.

In the capital goods sector, the value of constant capital is 0 and labor is  $L_1$  and product value is  $t_1\delta K$ . In the consumption goods sector, the value of constant capital is  $t_1\delta K$  and labor is  $L_0$  and product value is  $t_0Y$ .

Therefore, reproduction schema is as follows.

表 1: Reproduction schema(Price system)

	C	V	M	
capital goods sector		$wL_1$		$p_1\delta K$
consumption goods sector	$p_1\delta K$	$wL_0$		$p_0Y$

表 2: Reproduction schema(Value system)

	C	V	M	
capital goods sector		$L_1$		$t_1\delta K$
consumption goods sector	$t_1\delta K$	$L_0$		$t_0Y$

## 2. An extended model(Uzawa model)

In the basic model ,the capital goods are produced only by labor. But this assumption is so strong. Therefore we generalize this model in the case that capital goods are produced by labor and capital goods.In this case,capital goods is ultimately produced only by labor in the same way.

we suppose that capital goods  $K_0$  is produced by capital goods  $K_1$  and labor  $L_1$ .In the same way,we suppose that capital goods  $K_i$  is produced by capital goods  $K_{i+1}$  and labor  $L_{i+1}(i = 0, 1, 2, \dots)$ .In addition,we denote by  $A \leftarrow B$  that "A is produced by B". Then,

$$K_0 \leftarrow K_1 + L_1$$

$$K_1 \leftarrow K_2 + L_2$$

$$K_2 \leftarrow K_3 + L_3$$

.....

$$K_i \leftarrow K_{i+1} + L_{i+1}$$

.....

Consequently,

$$K_0 \leftarrow K_1 + L_1 \leftarrow K_2 + L_2 + L_1 \leftarrow K_3 + L_3 + L_2 + L_1 \dots \leftarrow K_i + L_i + \dots + L_2 + L_1 \leftarrow \dots$$

This indicates that capital goods is ultimately produced only by labor.

And now,we shall establish this model.The allocation of labor L is divided into the production of consumption goods and capital goods in the ratio  $L_0 : L_1$  ( $L_0 + L_1 = L$ ).In the same way,the allocation of capital goods K is divided into these two goods in the ratio  $K_0 : K_1$  ( $K_0 + K_1 = K$ ). We assume that the production functions for the first and second sector are of Cobb-Douglas type.

$$Y = AK_0^\alpha(L_0)^{1-\alpha} \equiv F(K_0, L_0) \tag{15}$$

$$\dot{K} = BK_1^\beta L_1^{1-\beta} - \delta K \equiv G(K_1, L_1) - \delta K \tag{16}$$

$\delta$  signifies the constant depreciation rate.

We set  $p_0, p_1, w, R, r$  in the same way in the previous section.

We suppose that capitalists only invests,labors only consume. That is,

$$p_0 Y = wL \tag{17}$$

$$p_1(\dot{K} + \delta K) = RK \tag{18}$$

Following equations hold under these setting.

$$p_0 Y_K = R \quad (19)$$

$$p_0 Y_L = w \quad (20)$$

$$p_1 G_K = R \quad (21)$$

$$p_1 G_L = w \quad (22)$$

The followings holds in the same way in the previous section.

$$\frac{K}{L} = \alpha \beta^{\frac{\beta}{1-\beta}} \left( \frac{B}{\delta} \right)^{\frac{1}{1-\beta}} \quad (23)$$

Reproduction schema is as follows in the same way.

表 3: reproduction schema(price system)

	C	V	M	
capital goods sector	$p_1 \delta K_1$	$w L_1$		$p_1 \delta K$
consumption goods sector	$p_1 \delta K_0$	$w L_0$		$p_0 Y$

表 4: reproduction schema(value system)

	C	V	M	
capital goods sector	$t_0 \delta K_1$	$L_1$		$t_1 \delta K$
consumption goods sector	$t_1 \delta K_0$	$L_0$		$t_0 Y$

## Conclusions

This reproduction schemas in section 1,2 imply the followings.

- 1.Profit is 0 in both sectors.
- 2.Surplus values are 0 in both sectors.
- 3.The exploitation rate  $\frac{M}{V}$  is 0 .
- 4.relative price is equal to relative value.(Because of  $\frac{p_0}{w} = t_0$ ,  $\frac{p_1}{w} = t_1$ )

That is,under the condition that capitalists only invest and labors only

consume, Marxian labor theory of value holds, supposed that capital goods are produced not only by labor but also by capital and labor. Uzawa called his 2 sectors model Marx= von Neuman model. But Marx= von Neuman model describes not so much capitalism as communism, because there doesn't exist exploitation and price equals value.

## References

[1] Shinkai, Y. (1960), "On the Equilibrium Growth of Capital and Labor," *International Economic Review*, Vol. 1, pp. 107-111

[2] Uzawa, H. (1961), "On a Two-Sector Model of Economic Growth," *Review of Economic Studies*, Vol. 29, pp. 40-47. Reprinted in *Preference, Production, and Capital: Selected Papers of Hirofumi Uzawa*, New York, Cambridge University Press, 1988

[3] Yamashita, Y. and Ohnishi, H. (2005), "A Marxist=Neo-classical Modeling of Capitalism as An Optimal Roundabout Production System," Kyoto university working paper No. 79

[4] Yamashita, Y. (2005), "Roemer's Exploitation in the Neo-classical "Marxist Model" of Growth," *Political Economy Quarterly* (Kikan Keizairiron), in Japanese

[5] Yuuho Yamashita, Hiroshi Ohnishi and Roxangul Wufurer (2004), "Reconstructing Marxism as a Neoclassical Optimal Growth Model," *Economic Study of Shanghai School*, Vol. 11, in Chinese

[6] Da Xiguang (Hiroshi Ohnishi) and Yin Luanyu (2005), "The Historical Materialism Analysis of Capital Accumulation," *Review of Political Economy*, vol. 8, no. 1, in Chinese.