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His books include: “financial relationship among governments in modern China” in 2001; “solution to the century difficulty——history and research on the transformation problem of value transformed into the price of production”

The Relations of Labor and Capital during Transformation and Real Wage Vector Problem

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I. Transformation of Labor Values into Prices of Production

The difficulty in the issue of transformation is how to prove the existence of general transformation model which satisfies "total agreement 2 proposition"(viz. total average profit equals to total surplus value, total production price equals to total value).It is not easy to deny the issue of transformation ,since it is really hard to prove inexistence of the transformation model. But it is enough to find one transformation model to affirm the issue, despite there may be several such models.

BSZ Transformation Model has supplied such a proof for the issue of transformation[1].

The basic thought of BSZ Transformation Model is as follows

Value system is:

$$\sum_{i=1}^{n} c_{ij} + v_j + m_i = w_j \qquad (j = 1,2,\cdots,n) \tag{1-1}$$

Production price system is:

$$(1+r)(\sum_{i=1}^{n} C_{ij} + V_j) = P_j \qquad (j = 1,2,\cdots,n) \tag{1-2}$$

1 The model was first addressed on the institution of theoretical economics in Japan in October, 2000.In Mar, 2005, Prof. Zou Zhongdan, department of maths in Bejing Normal University, had helped me solve the problem of condition of the model（see Huan Zhongdan and Zhang Zhongren, *A Necessary and Sufficient Condition of Positive Solutions to the BSZ Transformation Model*, Shimane Journal of Policy Studies [J], Vol.9, March 2005）.The model was standardized on the fist forum of Institution of International Political Economy in April,2006.In March of 2007, Prof Rong Zhaoxin, School of Economics in Anhui University found the model didn't need the condition of equivalency of the rate of surplus value. Thus, the model can be applied in extended area.

The relations of value system and production price system can be presented by BSZ Transformation Model as follows,

$$\begin{aligned}
&(1+r)(\sum_{i=1}^{n} x_i c_{ij} + y v_j) = x_j w_j \qquad (j = 1,2,\cdots,n) \\
&\sum_{j=1}^{n} x_j w_j = \sum_{j=1}^{n} w_j \qquad (1\text{-}3) \\
&\sum_{j=1}^{n} (\sum_{i=1}^{n} x_i c_{ij} + y v_j) = \sum_{j=1}^{n} (\sum_{i=1}^{n} c_{ij} + v_j)
\end{aligned}$$

Show in matrix

$$\begin{aligned}
&(1+r)(\mathbf{xc} + y\mathbf{v}) = \mathbf{x}\hat{\mathbf{w}} \\
&\mathbf{x}\mathbf{w}' = \mathbf{E}\mathbf{w}' \qquad (1\text{-}4) \\
&(\mathbf{xc} + y\mathbf{v})\mathbf{E}' = (\mathbf{Ec} + \mathbf{v})\mathbf{E}'
\end{aligned}$$

Condition of BSZ transformation model is $(1+r)\sum_{i=1}^{n} c_{ij} < w_j \qquad (j = 1,2,\cdots,n)$ [1],its key

$$\begin{aligned}
&r^* = \frac{\sum_{j=1}^{n} m_j}{\sum_{j=1}^{n} (\sum_{i=1}^{n} c_{ij} + v_j)} = \frac{m}{c+v} = \frac{w}{c+v} - 1 \\
&y^* = \frac{\mathbf{w}\mathbf{E}'}{\mathbf{v}(\frac{1}{1+r^*}\hat{\mathbf{w}} - \mathbf{c})^{-1}\mathbf{w}'} = \frac{w}{\mathbf{v}\left[\left(\frac{c+v}{w}\right)\hat{\mathbf{w}} - \mathbf{c}\right]^{-1}\mathbf{w}'} \qquad (1\text{-}5) \\
&X^* = y^*\mathbf{v}(\frac{1}{1+r^*}\hat{\mathbf{w}} - \mathbf{c})^{-1} = \frac{w\mathbf{v}\left[\left(\frac{c+v}{w}\right)\hat{\mathbf{w}} - \mathbf{c}\right]^{-1}}{\mathbf{v}\left[\left(\frac{c+v}{w}\right)\hat{\mathbf{w}} - \mathbf{c}\right]^{-1}\mathbf{w}'}
\end{aligned}$$

The key of BSZ transformation model satisfies “total agreement 2 proposition”.

After transformation, when $y = 1$, it needs no explain. But when $y \neq 1$,it means benefit of labor has changed, well then, how to apprehend such change? I think, it should be thought as transformation of labor values into prices of production.

As a kind of merchandize, when all goods transform into prices of production, labor should transform into prices of production, too. “y” means the deviation between production price of labor and labor value.

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1 make $b_{ij} = \frac{c_{ij}}{w_j}, \mathbf{b} = (b_{ij})_{n\times n}$,the condition can be put into the form of spectral radius $\rho(\mathbf{b}) < \frac{1}{1+r}$

II. Real Wage Vector of Samuelson and the Iron Law of Wages by Lassalle

When $y \neq 1$, a new problem arises, whether the real wage changes after transformation[1].If it does, and the change exists currently. Thus, it would be questionable about the existence of static transformation model, unless some new explanation comes out. If the change is conditional, we must illustrate the condition.

Samuelson (1957) suggested a method about real wage. We don't discuss the essence of the method here[2], but as a mathematical way to present, it should has two points. Firstly, unitize the value of merchandize, namely divide both sides of the formulas of Value system （1-1）and Production price system（1-2）by quantity q_i .Then we get

Unit of value system:

$$\sum_{i=1}^{n} c_{ij} / q_j + v_j / q_j + m_i / q_j = w_j / q_j \qquad (j = 1,2,\cdots,n) \quad (2\text{-}1)$$

Let $a_{ij} = c_{ij} / w_i q_j$, we make it be in the form of matrix as follows

$$\overline{\mathbf{w}}\mathbf{A} + \overline{\mathbf{v}} + \overline{\mathbf{m}} = \overline{\mathbf{w}} \qquad (2\text{-}1)'$$

Unit of Production price system:

$$(1+r)(\sum_{i=1}^{n} C_{ij} / q_j + V_j / q_j) = P_j / q_j \qquad (j = 1,2,\cdots,n) \quad (2\text{-}2)$$

Let $a_{ij} = C_{ij} / P_i q_j$, it could be made into form of matrix as follows

$$(1+r)(\overline{P}\mathbf{A} + \overline{\mathbf{V}}) = \hat{\overline{P}} \qquad (2\text{-}2)'$$

Pay attention, here, a_{ij} is considered invariable. But, we should notice that $(a_{ij})_{n\times n}$ is different from the input coefficient matrix we often know[3].

Samuelson expressed the real wage as a vector[4], we write it down as $\mathbf{d}' = (d_1, d_2, \cdots, d_n)$.

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1 The problem has been considered during the meeting on the fist forum of Institution of International Political Economy in April,2006.In March of 2007, Prof Rong Zhaozi in Anhui University and Prof Yu Bin in Peking University had discussed it with me successively. I could not affirm the DOF of subspace of real wage vector, so I was worrying how to explain if it was a narrow-road condition.

2 About essence of the method use by Samuelson .Please refer to Zhang Zhongren, issue of transformation: A circumstantial evidence for Marx supplied by Samuels on Economic study of Shanghai School ,2007(15).

3 After taking out the value or production price, it shows the pure physical quantity; but the input coefficient matrix(table of value)of Leontiefleft ratio of relative price.Besides,the numerical value is aslo different,since Samuelson divided by quantity in column ,and divided by value or production price in row, but Leontief just divided in column.

4 It was m in the original paper of Samuelson.

The method was good, but it was wrong for Samuelson to define real wage as the minimum necessaries. It showed that Samuelson did not grasp the wage theory of Marx, just fellow the iron law of wages by Lassalle[1].

In "wage、price and profit" published two years earlier than "Capital", Marx definitely pointed out that labor value" it was not a constant, but a variable, it was still a variable even though the value of other goods did not change[2].

The paper also expresses real wage as vector $\mathbf{d}' = (d_1, d_2, \cdots, d_n)$, but cancel the assumption of minimum necessaries.

Samuelson mistakenly thought unit of production price system(2-2) and unit of value system(2-1) were decided independently[3].They were two systems that could be substituted but not reconcilable.

III. Labor Force, Subordinate Merchandize?

After expressing real wage as vector $\mathbf{d}' = (d_1, d_2, \cdots, d_n)$, the problem whether labor force was independent variable appears. If labor force is not an independent variable, will it mean that labor force is subordinate merchandize?

In order to explain it, let us do some analysis. let us denote wage rate of value δ(inflation was discarded),so $\delta = \mathbf{wQ}^{-1}\mathbf{d}\,.\,\mathbf{wQ}^{-1}$ was cost vector unit. Denote labor force vector $\mathbf{L}$,so variable vector could be denoted $\mathbf{v} = \delta\mathbf{L} = \mathbf{wQ}^{-1}\mathbf{dL}$ .Considering $\mathbf{w}$ be $\mathbf{xw}$ after transformation, we reduce a variable y,so BSZ transformation model becomes as follows,

$$\begin{aligned} &(1+r)(\mathbf{xc} + \mathbf{x\hat{w}Q}^{-1}\mathbf{dL}) = \mathbf{x\hat{w}} \\ &\mathbf{xw}' = \mathbf{Ew}' \end{aligned} \qquad (3\text{-}1)$$

But we can't make sure that the key satisfies "total agreement 2 proposition". We talk about it later although it is vital. Now, after enactment of wage vector, wage rate of value is

1 In March ,1875, in "the letter to Au.Bebel" ,Engels animadverted the iron law of wages by Lassalle:"the basic of the law was hackneyde economial viewpint,namely worker only got the lowest wage on average .The reason was that worker was too many according to Malthusianism(It was the argument of Lssalle)."See "anthology of Marx and Engels(Ⅲ) , Rennin publication,1972(P28).Marx once had the same opinion, like "wage labor and capital"(1849)(see "Anthology of Marx and Engels(Ⅲ), Rennin publication,1972(P361).) The iron law of wages by Lassalle was raised in 1863,which made us feel Lassalle follow something abandoned by Marx.

2 Anthology of Marx and Engels(Ⅱ), Rennin publication,1972,P200.

3 Here, what Samuelson saw was the independency of system of production price, so, the value was concealed deeply in the dense fog. All made people could not see the real features easily.

no longer an independent variable, which means labor force is not an independent variable. Does it make sense theoretically?

Labor force is a special kind of merchandize but can't be substituted completely. Wage rate of value is value unit of labor force. If wage rate of value is not independent, labor force is a kind of subordinate merchandize at most. Is it the status of labor force in market? We feel that the status of labor force should be higher than ordinary (material) goods.

The value of labor force of course can be reflected by the value of n types of goods (in necessaries could be showed as 0). On the contrary, we know that all the goods are the product of labor, the value of n types of goods could also be reflected by labor force (it is possible mathematically), in this way, only one variable leaves. Does it make sense this way?

I think the problem about reflection of labor force value by n types of goods and the decision on the value of goods are not on the same level or not synchronous.

Decision on the value of labor force and goods should be involved in; whereas problem about reflection of labor force value by n types of goods should be out of the process of producing.

Decision on the value of labor force and goods should be at the same time; problem about reflection of labor force value by n types of goods should be a subordinate problem.

I leave it here as a very important theoretical problem,

IV. An Adequately Broad Access

Now, let us step back and see whether it makes sense regarding labor force as a sort of subordinate merchandize. Here, it is a bit troublesome to analyze following model （3-1）.Let's unitize it like Samuelson ,then model（3-1）changes as follows

$$\begin{aligned} &(1+r)(\mathbf{x}\hat{\overline{\mathbf{w}}}\mathbf{A}+\mathbf{x}\hat{\overline{\mathbf{w}}}\mathbf{dl})=\mathbf{x}\hat{\overline{\mathbf{w}}} \\ &\mathbf{x}\overline{\mathbf{w}}'=\mathbf{E}\overline{\mathbf{w}}' \end{aligned} \qquad (4\text{-}1)$$

Model （4-1）can be changed as follows

$$\begin{aligned} &\mathbf{x}\hat{\overline{\mathbf{w}}}(\mathbf{A}+\mathbf{dl})\hat{\overline{\mathbf{w}}}^{-1}=\frac{1}{1+r}\mathbf{x} \\ &\mathbf{x}\overline{\mathbf{w}}'=\mathbf{E}\overline{\mathbf{w}}' \end{aligned} \qquad (4\text{-}1)'$$

The condition is $\rho(\mathbf{A})<\frac{1}{1+r}$.So it is seen that $\frac{1}{1+r}$ is the eigen value of matrix $\hat{\overline{\mathbf{w}}}(\mathbf{A}+\mathbf{dl})\hat{\overline{\mathbf{w}}}^{-1}$,and $\mathbf{x}$ is its eigenvector.

But, (4-1)'doesn't satisfy "total agreement 2 proposition", it needs the following constraint condition:

$$\begin{aligned} &\overline{\mathbf{w}}\mathbf{d} = \frac{V}{L} \\ &\mathbf{l}(\frac{1}{1+r}\mathbf{I} - \mathbf{A})^{-1}\mathbf{d} = 1 \end{aligned} \qquad (4\text{-}2)$$

V and L are scalars, V is total constant capital, L is total labor force. In the system of equations, endogenous d is equivalent to the unknown. To state strictly the system of equations is the constraint condition of real wage vector d[1].

The system of equations has n unknown quantities, two equations, so the degree of freedom is n-2 which is very loose. When n is big enough, it could be sailed that there are almost no constraints. I.E. the real wage vector is vector subspace. So it can explain the different consumption of labor as a result of historical, cultural factors under the same industrial structure and wages.

What we need to do is: finding the condition of getting non-negative key to (4-2).

V. Epilogue

We have found an adequately broad access to the transformation model of Samuelson and Marx. Once I thought the two models was communicated well, but it doesn't seem so easy now. If we acknowledge the transformation model of Samuelson, we have to accept labor force as subordinate merchandize, even getting rid of the iron law of wages by Lassalle .Is it tenable in theory? I can't predicate at a word, but one thing to be sure is that the issue of transformation is becoming clearer.

1 According to the constraint condition upwards, Samuelson model is put as follows using Samuelson symbols:

$$\begin{aligned} &\mathbf{X}\hat{\pi}\left[\frac{1}{1+r}\mathbf{I} - (\mathbf{a}_0\mathbf{m} + \mathbf{a})\right] \geq 0 \\ &\pi(\mathbf{X}' - \mathbf{E}) = 0 \end{aligned}$$

Its condition is:

$$\begin{aligned} &(1+r)\sum_{i=1}^{n} a_{ij} < \pi_j \qquad (j = 1,2,\cdots,n) \\ &\pi\mathbf{m} = \frac{V}{L} \\ &\mathbf{a}_0(\frac{1}{1+r}\mathbf{I} - \mathbf{a})^{-1}\mathbf{m} = 1 \end{aligned}$$