

and corporations have to degrade the level of salary because of financial restriction. If those corporations do not change their salary policies, sub-laborers will retreat from this market too, which will lead to further degradation of salary level until the disappearance of inner-system labor market. Consequently, institutional segmentation of labor market will inevitably be unified in outside-system labor market and the speed of this unification will be faster when outside-system labor market is more and more perfect.

IV. Conclusion

The segmentation of labor market is a worldwide phenomenon. However, institutional segmentation is peculiar one existing in transforming countries, especially in China. We should see negative influence of institutional segmentation, although we should admit the historic achievements made by this segmentation. We can foresee that along with the establishment of market economy, institutional segmentation of labor market will be banished or even it cannot be completely eliminated at present, the influence of institutional segmentation will be greatly diminished.

We should work on both inner-system and outside-system labor market to resolve the problem of multi-layer labor market. By observing a way from outside-system to inner-system, from easy to difficult, we should pay more attention on the regulation and management of labor markets, which are newly set up in order to build a good operational environment for labor markets. We also should conduct reformation and innovation on traditional income allocation system, system of registered residence, social insurance and employment to provide a better systematical environment for establishing unified labor market, providing au pair employment and reducing income difference. Through these ways, institutional segmentation of labor market can be ended and a new labor market, which is unified, open, equally competitive and normative, can be established.

[JP] YUUHO YAMASHITA

Contact Information:

Address: Dokkyo University 1-1,
Gakuen-cho, Soka-shi, Saitama
340-0042, Japan

Email Address: yamashita76@dokkyo.ac.jp

Yuuho Yamashita, lecturer of Department of Economics, Dokkyo University, got Master's degree in Economics in Kyoto University, March 2000. His research areas include

Endogenous Growth Theory, Macroeconomics, and Marxian Economics.

His recent published papers are as follows:

(1) “A Marxist = Neo-classical Modeling of Capitalism as An Optimal Roundabout Production System”, with Hiroshi Ohnishi, *Kyoto University Working Paper*, July 2005

(2) “Roemer’s Exploitation in the Neo-classical ‘Marxist Model’ of Growth”, *Kikan Keizai Riron(Political Economy Quarterly)*, Vol.42, No.3, October, 2005, in Japanese

(3) “Reconstructing Marxism as a Neoclassical Optimal Growth Model”, *Seikei Kenkyu*, Institute of Political Economy, No.78, 2002, with Hiroshi Ohnishi, in Japanese

(4) “On the Labor as the Primary Factor of Production in ‘Marxist Model’”, *Keizai Ronso*, Kyoto University, Vol.172, No.3, 2003, with Hiroshi Ohnishi, in Japanese

(5) “Reconstructing Marxism as a Neoclassical Optimal Growth Model” *Economics Study of Shanghai School*, Vol.11, 2004, with Hiroshi Ohnishi and Roxangul Wufuer, in Chinese

A Marxist Neo-classical Growth Model with Competitive Labor Market

[JP] Yuuho Yamashita

Abstract: In this paper, I present the model to investigate dynamics of asset differential between the rich and the poor. Under the assumption that labor force is exchanged through competitive labor market, the model can determine accumulating path of total capital in the society but not the individual path of capital accumulation. Namely, there exists indeterminacy. What we can say is that the agent who wants to be the capitalist becomes the capitalist actually.

I. Introduction

Yamashita and Ohnishi(2005) presents the basic model of capitalism as an optimal roundabout production system. This paper is written to reconsider Marxian economics within a framework of neoclassical optimal growth theory. One of the main claims of the paper is that there are relatively so many similarities between Marxian economics and modern economics than have been thought. For example, transitory dynamics of a simple growth theory which describes the accumulation path of capital could be regarded as one of historical stages, i.e., capitalism. Based on Yamashita and Ohnishi(2005), Yamashita(2005) extend the basic model to two agents economy, i.e., the rich(or might as well be called capitalist) and the poor(or worker). And then Yamashita(2005) reconsider the Roemer's CECP(The Class Exploitation Correspondence Principle) and concludes as follows.

Whether exploitation disappears or not in the long-run depends on the optional right of the poor agent. If the poor does not have a right to determine what ratio of labor power he allocate for accumulating his own capital, the rich agent allocate all labor power of the poor in the production of consumer goods. In this case, amount of capital that the poor owns does not grow from the initial period and exploitation remains forever. In contrast, if the poor has a right to allocate his labor power for his own accumulation, he accumulates capital to the same amount as the rich. In this situation, the optimal response of the rich to the behavior of the poor is to accumulate capital to the same amount as the poor will do. In other words, the rich does not accumulate capital beyond the amount which the poor aims at and will achieve someday. The rich exploits the poor only in the transitional period toward the steady state where differential of asset between two agents does not exist anymore. This conclusion depends on the assumption that all of increment of production of consumer goods stemmed from equalization of capital-labor ratio between the rich and the poor is distributed to the rich. This is the very definition of the exploitation. In real economy, however, labor force is exchanged in the labor market and wage is paid to the labor supply. So, it is better to reconsider the conclusion of Yamashita(2005) in the settings where there is the labor market. In this paper, I introduce labor market into the two agents model. In Yamashita(2005), equalization of capital labor ratio between the rich and the poor is achieved by rending capital from the rich to the poor. On the other hand, in this paper it is achieved through labor market. The main conclusion of this paper is as follows. If the supply of labor force for the equalization of capital-labor ratio between two production field(self production by the poor and production by the firm) is realized through the competitive labor market, that is, the wage is equal to marginal productivity, the only variable that the model pin down is the total amount of capital. In other words, the individual amount of capital and its path to the steady state are undetermined.

II. Model

1. Settings of the model

In the economy, there is rich agent who has much more capital at initial period than poor agent. Both types of agent have same technology in production of consumer goods and capital goods. I assume the production system as below.

$$Y_t = AK_t^\alpha L_1^{1-\alpha} \quad (1)$$

$$K_{t+1} = K_t + BL_2 \quad (2)$$

$$L_1 + L_2 = 1 \quad (3)$$

Equation (1) represents a production function of consumer goods and equation(2) represents a production function of capital goods¹. Equation (3) is a resourceconstraint of labor force. Here, individual endowment of labor force is normalizedto 1. In this roundabout production system, labor force is substantial factor ofproduction.

The only difference between the rich and the poor is the endowment of capital at time 0. It is assumed that

$$K_{R0} > K_{P0}, \quad (4)$$

where K_{R0} is initial endowment of capital for the rich and K_{P0} for the poor. In the economy, there is the firm which is capitalized by the rich. The firm produces consumer goods so as to maximize a profit. The firm employs the rich and the poor to produce consumer goods and then pays wage w_t to them at same wage rate. Finally, the firm pays rental rate r_t to the rich.

2. Behavior of the poor

The poor allocates his labor force to three different activities. The first (u_t) is used for the self production of consumer goods. The second (s_t) is used for the supply of labor force to the firm. The third ($1 - u_t - s_t$) is used for production of capital goods for his accumulation.

The poor choose u_t and s_t so as to maximize his life time utility. The poor's consumption at time t (C_{Pt}) is consisted of self production and wage. That is,

$$C_{Pt} = AK_{Pt}^\alpha (u_t)^{1-\alpha} + w_t s_t. \quad (5)$$

In addition, Capital stock of the poor grows according to equation (6).

$$K_{Pt+1} = K_{Pt} + B(1 - u_t - s_t). \quad (6)$$

It is assumed that lifetime utility of the poor is

1 For simplicity, the rate of depreciation is assumed to be zero.

$$U = \sum_{t=0}^{\infty} \beta^t C_{Pt}. \quad (7)$$

Therefore, Lagrange function of the poor is

$$L_P = \sum_{t=0}^{\infty} \beta^t [AK_{Pt}^{\alpha} (u_t)^{1-\alpha} + w_t s_t + \lambda_t \{K_{Pt+1} - K_{Pt} - B(1 - u_t - s_t)\}] \quad (8)$$

The first order conditions are

$$(1 - \alpha)AK_{Pt}^{\alpha} u_t^{-\alpha} + B\lambda_t = 0, \quad (9)$$

$$w_t + B\lambda_t = 0, \quad (10)$$

$$\alpha AK_{Pt}^{\alpha-1} u_t^{1-\alpha} - \lambda_t + \beta^{-1} \lambda_{t-1} = 0. \quad (11)$$

3. Behavior of the rich

The rich agent invests his capital to the firm and earns rental rate r_t as capitalist. On the other hand, he allocates his labor force into two activities. One (v_t) is allocated the supply of his labor force to the firm. The other $(1 - v_t)$ is allocated for production of his own capital. Here, I assume that utility function of the rich is same as the poor.

Finally, capital stock of the rich grows according to equation (12).

$$K_{Rt+1} = K_{Rt} + B(1 - v_t) \quad (12)$$

Therefore, Lagrange function of the rich is

$$L_R = \sum_{t=0}^{\infty} \beta^t [r_t K_{Rt} + w_t v_t + \gamma_t \{K_{Rt+1} - K_{Rt} - B(1 - v_t)\}] \quad (13)$$

The first order conditions are

$$w_t + B\gamma_t = 0, \quad (14)$$

$$r_t - \gamma_t + \beta^{-1} \gamma_{t-1} = 0. \quad (15)$$

4. Behavior of the Firm

A profit of the firm π is

$$\pi = AK_{Rt}^{\alpha} (s_t + v_t)^{1-\alpha} - r_t K_{Rt} - w_t (s_t + v_t). \quad (16)$$

The first order conditions for profit maximization are

$$(1 - \alpha)AK_{Rt}^{\alpha} (s_t + v_t)^{-\alpha} - w_t = 0, \quad (17)$$

$$\alpha AK_{Rt}^{\alpha-1} (s_t + v_t)^{1-\alpha} - r_t = 0. \quad (18)$$

5. Long-run equilibrium

In previous subsections, differentiation about profit maximization of the firm is done

with respect to $s_t + v_t$ and K_{Rt} taking prices (w_t, r_t) as given. The rich and the poor also take prices as given. This means that labor and capital markets are competitive.

The dynamics of the economy is represented by nine equations, (6), (9), (10), (11), (12), (14), (15), (17), (18) and initial conditions K_{P0}, K_{R0} . This simultaneous equation consists of has nine variables, $K_{Pt}, K_{Rt}, u_t, s_t, v_t, w_t, r_t, \lambda_t, \gamma_t$.

From (9), (10), (17), we can derive

$$K_{Pt}^\alpha u_t^{-\alpha} = K_{Rt}^\alpha (s_t + v_t)^{-\alpha} \quad (19)$$

Here, we define that

$$k_{Pt} \equiv \frac{K_{Pt}}{u_t}, \quad k_{Rt} \equiv \frac{K_{Rt}}{s_t + v_t}. \quad (20)$$

k_{Pt} and k_{Rt} are the capital-labor ratio in production of consumer goods at time t by the poor and by the rich, respectively. From (19) and (20),

$$k_{Pt} = k_{Rt}. \quad (21)$$

This means that capital-labor ratios of two production fields are equalized. By this condition, total amount of production of consumer goods in the whole society at time t is maximized.

From (9), (11) and (17), we can derive

$$\alpha\beta B k_{Pt}^{\alpha-1} + (1-\alpha)\beta k_{Pt}^\alpha - (1-\alpha)k_{Pt-1}^\alpha = 0. \quad (22)$$

capital-labor ratio of the poor's self production of consumer goods is subject to this difference equation. As is the case for the poor agent, the difference equation of capital-labor ratio of the firm's production is derived from (14), (15), (17) and (18) as follows.

$$\alpha\beta B k_{Rt}^{\alpha-1} + (1-\alpha)\beta k_{Rt}^\alpha - (1-\alpha)k_{Rt-1}^\alpha = 0. \quad (23)$$

(22) and (23) have same form. So, k_{Pt} and K_{Rt} have common steady state.

$$k_P^* = k_R^* = \frac{\alpha\beta B}{(1-\alpha)(1-\beta)} \equiv k^*. \quad (24)$$

Note that (24) means only the equalization of capital-labor ratios in two production fields and not the equalization of k_{Pt} and K_{Rt} . In other words, (24) has nothing to do with the ownership of capital.

6. Transitional Dynamics

Movement of k_{Pt} and K_{Rt} are governed by (22) and (23), respectively. Actually, k_{Pt} and K_{Rt} do not change over time. In other words, these take the value of k^* not only in the long-run steady state but also in a transitional path to the steady state. Intuitively speaking, this is because we assume the linear felicity function in (7). Linear felicity function means that utility maximization is the same thing to maximize the amount of production. To maximize the amount of production, it is necessary to keep the most

efficient capital-labor ratio over time. I think that we could justify this assumption of linear felicity function by the one of Kaldor’s stylized fact that the real rate of return on capital is basically constant. In competitive market, the real rate of return of capital is same as the marginal productivity of capital which corresponds one to one to the capital-labor ratio.

Therefore, we can see that

$$k_{Pt} = k_{Rt} = k^*, \text{ for all } t. \tag{25}$$

By the constantness of k_{Pt} and k_{Rt} , we can derive

$$u_t = \frac{K_{Pt}}{k^*} \tag{26}$$

$$s_t + v_t = \frac{K_{Rt}}{k^*} \tag{27}$$

From (6), (12), (26), (27), we have

$$(K_{Pt+1} + K_{Rt+1}) = 2B + \frac{k^* - B}{k^*}(K_{Pt} + K_{Rt}). \tag{28}$$

Here, define total amount of capital in the society.

$$X_t \equiv K_{Pt} + K_{Rt} \tag{29}$$

Therefore, from (28) and (29), we can derive

$$X_{t+1} = 2B + \frac{k^* - B}{k^*}X_t, \quad X_0 = K_{P0} + K_{R0} \text{ is given} \tag{30}$$

(30) is simple first order difference equation and we can draw figure 1 which describes transitional dynamics of X_t . In addition, (30) has a single steady state,

$$X^* = 2k^* = \frac{2\alpha\beta B}{(1 - \alpha)(1 - \beta)}, \tag{31}$$

From figure 1, we can see that the total amount of capital in the society converges to (31) in the long run.

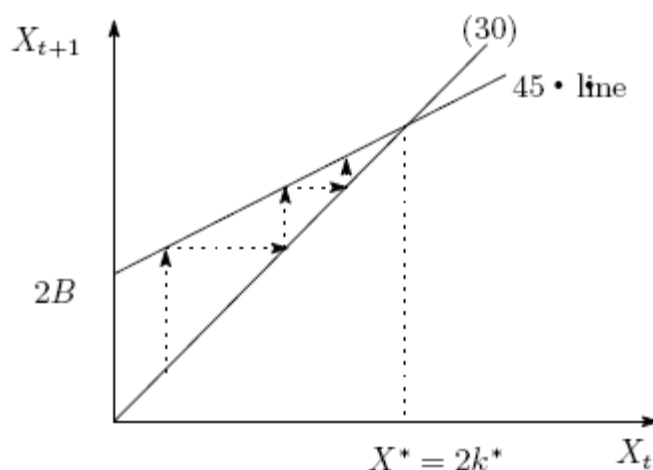


Figure 1

7. Indeterminacy of Individual Path

In the previous section, we saw how the total amount of capital in the society grows over time. In this section, the dynamics of individual capital is considered. There are five endogenous variables, u_t , s_t , v_t , K_{Pt} , and K_{Rt} . These variables are governed by the four simultaneous equations system and initial conditions. That is,

$$u_t = K_{Pt}k^{*-1} \quad (32)$$

$$s_t + v_t = K_{Rt}k^{*-1} \quad (33)$$

$$K_{Pt+1} = K_{Pt} + B(1 - u_t - s_t) K_{P0} : \text{given} \quad (34)$$

$$K_{Rt+1} = K_{Rt} + B(1 - v_t)K_{R0} : \text{given.} \quad (35)$$

By this system, we can find the optimal accumulating path of total amount of capital as did in previous section but we cannot find the determinate path of individual capital. What is the most important is the lack of conditions to pin down three control variables (u_t , s_t and v_t).

To pin down individual accumulating path, we have need to specify one of three control variables exogenously. For example, assume that v_t is determined at some level exogenously at period t . Necessarily, $1-v_t$ is determined. This means that the rich determines how much capital he accumulates. By the way, the optimal total amount of production of capital at time t has been determined already by (30). Then, the poor determines $1 - u_t - s_t$ to produce remaining part of capital accumulation, i.e., the optimal total amount of capital production at period t minus amount of capital production by the rich. Finally, the poor determines the ratio of labor supply to labor allocation for self production of consumer goods (s_t/u_t) in order to equalize capital-labor ratio between two production fields.

Intuitively, we can summarize why such indeterminacy arises as follows.

The rich and the poor can earn wage which is determined by marginal productivity of labor. This is because the firm maximizes the profit in competitive market. This means that if one agent accumulates the capital, wage rate increases and the other agent can earn more wage. As a matter of course, the agent who has more capital than the other can be capitalist and earn rental rate for his capital. To earn the rental rate, however, he must allocate his labor force to the production of capital goods which does not provide utility directly. That is to say, to earn the rental rate, or stated another way, to become the capitalist, he must give up some wage income. In labor market equilibrium, it is indifference for agents to which agent becomes capitalist, because marginal utilities gained from two activities, one has roots in wage and the other has roots in rental rate, are equalized. The only advantage of the rich is that he has much more capital than the poor at initial period.

By the above discussion, we can understand that the asset differential between agents may increase or may decrease. We cannot forecast which occur actually. In both case, however, there is no difference in lifetime utility level without the one which arises from the initial asset inequality.

We can say that the agent who wants to be the capitalist becomes the capitalist and the other becomes the worker.

III. Concluding Remarks

The main conclusions of this paper are as follows.

If labor forces are exchanged through competitive labor market for the equalization of capital-labor ratio between two production fields (self production by the poor and production by the firm), that is, wage is equal to marginal productivity, the only variable that the model pin down is the total amount of capital. In other words, the individual amount of capital and its accumulation path to the steady state are undetermined.

I do not claim that the model of this paper is realistic. But I think the model represents one aspect of the real economy. All of the people in the society could be benefited from capital accumulation regardless whether he owns the capital or not.

From the point of view of Roemer's exploitation, there is much smaller exploitation than the model of Yamashita (2005) where it is assumed that all of increment of production of consumer goods stemmed from equalization of capital-labor ratio is distributed to the rich.

The conclusions of the paper depend on the assumption that labor market is competitive. This assumption can be justified implicitly by the existence of huge number of rich and poor agents in the economy. From the perspective of Marxian economics, however, we may have to consider the scarcity of capital at least early stage of capitalism. That is to say, there are so much number of poor and a small number of capitalists. In this case, the competitiveness of labor market cannot be justified and capitalists can behave as monopolist. In other words, the rich agent takes effect of his labor supply on wage into account when he maximizes his lifetime utility. This asymmetry is left for further research.

References

[1] Yamashita, Yuuho and Hiroshi Ohnishi., A Marxist=Neo-classical Modeling of Capitalism as An Optimal Roundabout Production System', Kyoto university Working paper, No.79, July 2005.

[2] Yamashita, Yuuho, Roemer's Exploitation in the Neo-classical"Marxist Model"of

Growth', Kikan Keizai Riron (Political Economy Quarterly), Vol.42, No.3, 2005, in Japanese.

[3] Yamashita, Yuuho and Hiroshi Ohnishi. Reconstructing Marxism as a Neoclassical Optimal Growth Model Seikei Kenkyu, Institute of Political Economy, No.78, 2002, in Japanese.

[4] Yamashita, Yuuho and Hiroshi Ohnishi. On the Labor as the Primary Factor of Production in "Marxist Model", Keizai Ronso, Kyoto University, Vol.172, No.3, 2003, in Japanese.

[5] Yamashita, Yuuho, Hiroshi Ohnishi and Roxangul Wufuer. Reconstructing Marxism as a Neoclassical Optimal Growth Model, Economics Study of Shanghai School, Vol.11, 2004, with Hiroshi Ohnishi and Roxangul Wufuer, in Chinese.

[JP] RYO KANAE

Contact Information:

Email Address: kanaeryo@yahoo.co.jp

Ryo Kanae is a student of the graduate school of Kyoto University.