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The Neoclassical Marxian Model

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I. Introduction

In Yamashita-Ohnishi(2005 [1]) and Yamashita(2006 [2]),Marxian economics is reinterpreted in terms of the optimal growth theory. They assumed that capital goods are produced only by labor force. In section 1,we outline this basic model. In section 2,we generalize this model to the case that capital goods are produced by labor force and capital goods. In section 3,we consider the market equilibrium in the basic model.

II. The Basic Model

We assumed that society has two sectors of production— the consumption goods sector (the first sector) and the capital goods sector (the second sector). Factors of production are capital goods and labor. Consumption goods are produced by capital goods and labor, and capital goods is only labor. Consumption goods is ultimately only created by labor (The Labor theory of value).

We denote by Y, K, L, respectively, consumption goods, capital goods, labor .In addition, the allocation of labor L is divided into these two sectors in the ratio s: 1-s. We assume that the production function for the first sector is of Cobb-Douglas type.

$$Y = AK^{\alpha}(sL)^{1-\alpha}$$

Furthermore, we assume that the production function for the second sector is simply as follows.

$$\dot{K} = B(1-s)L \quad (1)$$

$\dot{K}$ denotes a differential with respect to time.

The lifetime utility U of an individual is

$$U = \int_0^{\infty} e^{-\rho t} logY dt$$

ρ represents time preference, *logy* ,instantaneous utility at each point t.

The issue of optimization of society over time is to maximize U subject to (1).

The current value Hamiltonian H is

$$H = logY + \lambda B(1-s)L$$

$$= logA + \alpha logK + (1-\alpha)logs + (1-\alpha)logL + \lambda B(1-s)L$$

λ is shadow price of capital goods. The first order conditions are as follows:

$$\frac{\partial H}{\partial s} = 0 \Longleftrightarrow \frac{1-\alpha}{s} - \lambda BL = 0 \quad (2)$$

$$\frac{\partial H}{\partial K} = \rho\lambda - \dot{\lambda} \Longleftrightarrow \frac{\alpha}{K} = \rho\lambda - \dot{\lambda} \quad (3)$$

Here, we define the growth rate of a variable.

The growth rate of x is defined as follows.

$$\hat{x} := \frac{\dot{x}}{x}$$

We will obtain the long-term equilibrium.

From (2)

$$log(\frac{1-\alpha}{s}) = log(\lambda BL).$$

Differentiating this equation with respect to time,

$$-\hat{s} = \hat{\lambda} \quad (4)$$

Dividing (3) by λ,

$$\frac{\alpha}{\lambda K} = \rho - \hat{\lambda}$$

Since the growth rate $\hat{\lambda}$ is constant in the optimal steady state, the right hand side of this

equation is constant.

Hence,

$$\hat{\lambda} + \hat{K} = 0 \quad (5)$$

Since $\hat{s} = 0$ in the long term equilibrium, $\hat{K} = 0$ is obtained from (4); (5). This means that economic growth will be stopped in some day and capitalism will disappear.

III. An Extension of the Basic Model

In the basic model, the capital goods are produced only by labor. But this assumption is so strong. Organic composition of capital in the capital goods sector is usually larger than that in the consumption goods sector. Therefore we generalize this model in the case that capital goods are produced by labor force and capital goods. In this case, capital goods is ultimately produced only by labor in the same way.

For example, we suppose that capital goods K_0 is produced by capital goods K_1 and labor L_1.In the same way, we suppose that capital goods Ki is produced by capital goods K_{i+1} and labor L_{i+1}($i = 0,1,2, \ldots$).In addition, we denote by $A \leftarrow B$ that "A is produced by B". Then,

$K_0 \leftarrow K_1 + L_1$

$K_1 \leftarrow K_2 + L_2$

$K_2 \leftarrow K_3 + L_3$

……………

$Ki \leftarrow Ki+1 + Li+1$

……………

Consequently,

$K_0 \leftarrow K_1 + L_1 \leftarrow K_2 + L_2 + L_1 \leftarrow K_3 + L_3 + L_2 + L_1 \cdots \leftarrow K_i + L_i + \cdots + L_2 + L_1 \leftarrow \cdots$

This indicates that capital goods is produced only by labor.

And now, we will establish this model. The allocation of labor L is divided into the production of consumption goods and capital goods in the ratio $s1 : s2$ ($s1 + s2 = 1$).In the same way, the allocation of capital goods K is divided into these two goods in the ratio $s_1 : s_2 \quad (s_1 + s_2 = 1)$. We assume that the production functions for the first and second sector are of Cobb-Douglas type.

$$Y = A(\phi_1 K)^\alpha (s_1 L)^{1-\alpha}$$

$$\dot{K} = B(\phi_2 K)^\beta (s_2 L)^{1-\beta}$$

The lifetime utility U of an individual is

$$U = \int_0^\infty logY dt$$

$$s_1 + s_2 = 1, \phi_1 + \phi_2 = 1$$

The current value Hamiltonian H with the constrained conditions is

$$\begin{aligned} H &:= logY + \lambda\dot{K} + \mu(1 - \phi_1 - \phi_2) + \epsilon(1 - s_1 - s_2) \\ &= logA + \alpha log\phi_1 + \alpha logK + (1-\alpha)logs_1 + (1-\alpha)logL + \lambda B(\phi_2 K)^\beta (s_2 L)^{1-\beta}. \end{aligned}$$

The first conditions in this model are as follows.

$$\frac{\partial H}{\partial \phi_1} = 0, \frac{\partial H}{\partial \phi_2} = 0 \Longleftrightarrow \frac{\alpha}{\phi_1} = \lambda B\beta(\phi_2 K)^{\beta-1}(s_2 L)^{1-\beta} K \qquad (6)$$

$$\frac{\partial H}{\partial s_1} = 0, \frac{\partial H}{\partial s_2} = 0 \Longleftrightarrow \frac{1-\alpha}{s_1} = \lambda B(1-\beta)(\phi_2 K)^\beta (s_2 L)^{-\beta} L \qquad (7)$$

$$\frac{\partial H}{\partial K} = \rho\lambda - \dot{\lambda} \qquad (8)$$

From (6),

$$-\hat{\phi_1} = \hat{\lambda} + (\beta - 1)\hat{\phi_2} + (\beta - 1)\hat{K} + (1-\beta)\hat{s_2} + \hat{K}$$

By the way, $\hat{\phi_1} = \hat{\phi_2} = \hat{s_1} = \hat{s_2} = 0$ in the optimal steady state. Hence,

$\hat{\lambda} + \beta\hat{K} = 0.$ (9)

We obtain (9) in the same way from (7).

$$\begin{aligned} &\tfrac{\partial H}{\partial K} = \rho\lambda - \dot{\lambda} \\ &\Longleftrightarrow \tfrac{\alpha}{K} + \lambda B\beta(\phi_2 K)^{\beta-1}(s_2 L)^{1-\beta}\phi_2 = \rho\lambda - \dot{\lambda} \\ &\Longleftrightarrow B\beta(\phi_2 K)^{\beta-1}(s_2 L)^{1-\beta}\phi_2 = \rho - \hat{\lambda}(see(6)) \end{aligned}$$

Since the right hand side of this equation is constant in the long term equilibrium,

$$(\beta-1)\hat{\phi_2}+(\beta-1)\hat{K}+(1-\beta)\hat{s_2}=0$$

$(\beta-1)\hat{K}=0$,hence $\hat{K}=0$.

This means that economic growth will be stopped in some day and capitalism will disappear.

IV. The Market Equilibrium

In this section, we consider the market equilibrium in the basic model.

The consumption goods firm: $Y=AK^{\alpha}(L_1)^{1-\alpha}$

The capital goods firm: $\dot{K}=BL_1$

Labor market: $L1 + L2 = L$

The Lifetime Utility of an individual: $U=\int_0^{\infty}e^{-\rho t}logY dt$

We let notations as follows:

The price of consumption goods per unit:1

The price of capital goods per unit: p

The interest rate: r

The wage rate of the consumption goods firm: w_1

The wage rate of the capital goods firm: w_2.

The profit of two kinds of firms is as follows.

The profit of the consumption goods firm: $\pi=1\cdot Y-rpK-w_1L_1$

The profit of the capital goods firm: $\Pi=p\dot{K}-w_2L_2$

We denote the asset by A. The asset of household is $A = pK$.

The budget constraint of household: $\mathcal{A}=r\mathcal{A}+w_1L_1+w_2L_2-Y+\dot{p}K$

In the right hand side of this equation, $\dot{p}K$ means capital gain.

The first order conditions of the capital goods firm are as follows.

$\frac{\partial\pi}{\partial K}=0\Leftrightarrow Y_K=rp$

$\frac{\partial\pi}{\partial L_1}=0\Leftrightarrow Y_{L_1}=w_1$

Consequently, $\pi=0$.

The profit of the consumption goods firm is $\Pi=p\dot{K}-w_2L_2=(pB-w_2)L_2$. If $pB>w_2$, then $L_2=L,L_1=0$. If $pB<w_2$ then $L_2=0,L_1=L$. We consider the case of $pB=w_2$.

The equilibrium condition of the labor market is $w1 = w2$. This means that wage rates of the two kinds of firms aren't different.

In this model, society has four markets. That is, society has markets of consumption goods, capital goods, labor, and money. Among these four markets, Walrus'' law is realized.

$$(Y - rpK - w_1L_1) + (p\dot{K} - w_2L_2) + (w_1L_1 + w_2L_2 - wL) + (r\mathcal{A} + wL - Y + \dot{p}K - \dot{\mathcal{A}}) = 0$$

Household maximizes the Lifetime Utility

$$U = \int_0^\infty e^{-\rho t} logY\,dt.$$

In the optimal steady states,

$$\frac{\dot{Y}}{Y} = r - \rho = 0.$$

r comes near to ρ.In the optimal steady states, $r = \rho$

In this optimal states, $Y_K = rp = \rho p$,that is, $p = \frac{Y_K}{\rho}$.

By substituting this equation into $pB = w$, we obtain

$$\frac{1}{\rho} Y_K B = w.$$

From this equation and $\dot{K} = BL_1$, $Y_{L_1} = w$,we obtain

$$\frac{1}{\rho}\frac{\partial Y}{\partial K}\frac{\partial K}{\partial L} = \frac{\partial Y}{\partial L}$$

This means that social optimization and market equilibrium aren't different in the Neoclassical Marxian Model.

V. Conclusions

(1) We can maintain that it is possible to interpret Marxian value theory from the viewpoint of a neo-classical growth theory in the case of capital goods are produced by capital goods and labor.

(2) In the Neoclassical Marxian Model, social optimization is equal to market equilibrium.

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