

compensated by public article that is unceasingly invested.

## V. The Creation of the Wealth and the Distribution of the Income

The viewpoint of the Socialism public ownership system, is the perfect behavior that is complied with the objective economic law and is satisfied the request of the productive forces development, and is the rightful call that realizes the society to be fair and promotes the general working people to be benefit. The truth and fact of labor creating value, labor creating wealth objectively requests all the value or the wealth are held by all working people, and implementing public ownership system. The country that has not been able to realize the public ownership system, as well as the public ownership system country incapably realizes the domain of the public ownership system, must control the status of the enterprise leadership to limit the bourgeois benefit in the essential scop , for instance the scope which attracts the investment needs.

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3. "Economic Analysis on Information Goods (Jouhou-zai no Keizai Bunseki in Japanese)" (2005), Showa-do.
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## **Updating Valuations and Stochastic Dynamics on Coordination ——Implications on Capital-Labor Relations**

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### **I. Introduction**

Relationship between labor and capital is one of important factors that affect on efficiency of production. Especially, under great industry by machinery, the amount and quality of machine, i.e. capital, is the dominant factor for efficiency. This brings strong bargaining power to capital. However, since 1980's, information industry and soft economy have been developed. Eventually, labor can accumulate a human capital. And they also have some bargaining power against capital. In addition to this, in information industry and soft economy, some coordination among producers including capital and labor are important in the process of production. This kind of coordination is not resolved by the dictatorial power of someone over others, e.g. capital and labor. This is because each producer within a firm has some specialized knowledge and some bargaining power. In this

paper, we examine evolutionary consequences of coordination. With these results, we try to derive some implications for the coordination within a firm in “Post-Capitalism” defined by Ohnishi (2004).

In the production procedures, it is not possible to know everything relevant when we face decisions. Generally, the limited knowledge we have is based on personal experience. If someone succeeds in gaining relatively higher payoff, he/she would retain that choice of action. Such subjective valuations of actions are critical factors in maintaining stable situations. Of course, the outcome of an action depends on the other person’s action, and vice versa.

In this paper, under such patterns of behavior, we examine the equilibrium selection in a symmetric coordination game. We assume that each player has valuations of possible actions. If an action is chosen, the valuation of the action is updated according to the sign of the difference between current payoff and current valuation of the action. If the realized payoff of the action exceeds the valuation then the valuation increases; otherwise it decreases. If both are equal, it does not change. If the valuation of the current action exceeds the valuation of the alternative, the player chooses the same action next time. Also, because of the perturbation, the player chooses the undesirable action with small probability.<sup>1</sup>

This behavior rule shares basic features with satisficing behavior (Karandikar, Mookherjee, Ray and Vega-Redond 1998, In-Koo and Matsui 2005, Kim 1999, Pazgal 1997). The features of satisficing behavior are summarized as follows: (1) there is *one* endogenous parameter (aspiration) that is updated, based on realized payoffs, and converges to the realized payoffs.<sup>2</sup>(2) players use the parameter to trigger changes of action. In the present model we have *two* endogenous parameters (valuations for actions), of which one converges to the realized payoffs. Regarding (2), two endogenous parameters (valuations for actions) are used in turn as a reference point.

In the present model, the Risk dominant outcomes are selected in the long run in the coordination game irrespective of the initial conditions. This is in contrast to the situation in (Karandikar et al. 1998, Pazgal 1997, Kim 1999), in which the Pareto efficient outcome is selected under certain conditions.<sup>3</sup>

This difference stems from the number of endogenous reference points. In the other papers, the endogenous reference point is the aspiration level. Based on this reference point,

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1 This trembling for choice is also assumed by In-Koo and Matsui (2005) and Pazgal (1997). Conversely, Karandikar et al (1998) assume that aspiration is perturbed directly.

2 In Pazgal (1997), the endogenous parameter (aspiration) converges to the maximal payoff in past experience.

3 In Pazgal (1997), Kim (1999), an initial high aspiration level is needed. The results in Karandikar et al. (1998) do not depend on the initial conditions, but require sufficiently slow updating of aspiration.

the action is chosen. In the simple satisficing model (Karandikar et al. 1998), if the equilibrium outcome is Pareto efficient, the aspiration is higher than the Risk dominant outcome. It is therefore difficult to move to the Risk dominant outcome. On the other hand, if the equilibrium outcome is Pareto inefficient (and Risk dominant), the aspiration is lower than the Pareto efficient outcome and it is easy to move to the Pareto efficient outcome.

In case-based decision theory (Pazgal 1997, Kim 1999), valuations of actions are calculated at every time. The aspiration level is used as a reference point for the valuations, and there is a device to maintain the aspiration level high. In Pazgal (1997), it is assumed that aspiration converges to the maximum payoff in the past. Also, if the initial aspiration level is high enough, players are not satisfied with their current situation and make many experiments in the early periods. The Pareto efficient outcome then occurs with probability close to 1. In this situation the player is not satisfied with the Pareto inefficient outcome, generating an inclination to Pareto efficient outcomes.

In Kim (1999), let us suppose that the aspiration level is close to the efficient payoff and the adjustment speed is slow. If the efficient outcome is realized once, the outcome 3In Pazgal (1997), Kim (1999), an initial high aspiration level is needed. The results in Karandikar et al. (1998) do not depend on the initial conditions, but require sufficiently slow updating of aspiration.

Then continues. This is because, by the way of making valuations, positive values are accumulated to the valuation of the efficient action. On the other hand, even if the risk dominant outcome is realized, the valuation of the risk dominant action becomes less than that of the efficient action after a finite period. This is because, with a high aspiration level, the negative values accumulate to the valuation of the risk dominant action. As a result of this negative accumulation and slow adjustment, the valuation of risk dominant action decreases.

In contrast to previous research, players in our model use two reference points in turn. This weakens commitment to the efficient outcome. This weakening is because the reference point is a valuation of the other actions, and it cannot maintain a high value. Hence, the risk dominant outcome comes to be realized in the long run. When we focus more closely on behavior patterns, the differences are as follows. When a player quits the current action and chooses a new action, in Karandikar et al. (1998) and In-Koo and Matsui (2005), a very simple manner is assumed: all actions can be chosen with equal probability. In Kim (1999) and Pazgal (1997), a more sophisticated manner is assumed: players adopt an action that has the largest cumulative payoff rescaled by the current aspiration level. In that case, players have to remember all past experience in the past.<sup>4 1</sup> On the other hand, in the present situation, players update one of the valuations directly at every time, so that

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<sup>1</sup> There is a simple formula for levels of valuations. However, players have to remember the number of times players choose a particular action.

players need to remember only two numbers (the valuations of two actions). Of course, if the situation is more complex, it is too difficult to consider situations carefully at each choice of action. Karandikar et al. (1993) and In-Koo and Matsui (2005) are then helpful guides. If the situation is important for the players, they would adopt the methodology of Kim (1999) and Pazgal (1997). But when the situation is not overly complex and not overly important, the present methodology applies.

The rule of thumb in the present model shares common features with the reinforcement dynamics of Roth and Erev (1995) and Erev and Roth (1998). Under reinforcement dynamics there are two endogenous parameters, as here. In the present model, however, players use a pure strategy except for tie breaking. Under reinforcement dynamics, players use a mixed strategy and valuations of actions are used as the weight for mixing. Hence, in our model, players stick firmly to the current action so long as the valuations indicate that it is better. In other words, the inertia is stronger than the reinforcement dynamics.

In the coordination game, it is possible that players afraid of frequent changes of action would send complex signals to the other player. Consequently, a pure strategy assumption in this model may also be relevant.

The relationship among models is summarized in Table 1.

**Table 1: The present model, aspiration model and reinforcement model**

	One Endogenous Parameter	Two Endogenous Parameters
Pure Strategy	Aspiration Model	Present Model
Mixed Strategy		Reinforcement Model

In the next section we describe the model. Section 3 presents results. Section 4 sketches the proofs of the results. Section 5 gives concluding comments.

## II. The Model

Consider the following 2×2 game:

	$A$	$B$
$A$	$a; a$	$d; c$
$B$	$c; d$	$b; b$

In which we assume that  $a > c$ ,  $b > d$  and  $a + d < b + c$ . Both  $(A, A)$  and  $(B, B)$  are Nash equilibrium, and  $(A, A)$  is a Pareto efficient outcome and  $(B, B)$  is a risk dominant

outcome.

We also assume that players divided the continuous domain for evaluation into a same sized grid, and recognize only which sector the payoffs or valuations fall in. If we label all grids with positive integers, then  $a, b, c, d \in \mathbb{Z}^+$  where  $\mathbb{Z}^+$  denotes the set of positive integers. The players are indexed by  $i = 1, 2$ . Player  $i$ 's state at time  $t \geq 0$  is defined by two valuations, for two actions, as  $(v_{A,t}^i, v_{B,t}^i)$ . We take all valuations as positive without loss of generality.<sup>51</sup> Initial states are  $H, L \in \mathbb{Z}^+$  such that  $H > a$  and  $L < d$ , and for  $i \in \{1, 2\}$ ,  $v_{A,0}^i, v_{B,0}^i \in [L, H]$ . A (social) state  $s$  is the pair of states of Players 1 and 2. Thus, at time  $t$ , it follows that  $s \equiv [(v_{A,t}^1, v_{B,t}^1); (v_{A,t}^2, v_{B,t}^2)]$

Given Player  $i$ 's state at period  $t$ , Player  $i$  adopts the action that has higher valuation:

$$x_t^i = A \quad \text{if } v_{A,t} > v_{B,t} \tag{1}$$

$$x_t^i = B \quad \text{if } v_{A,t} < v_{B,t}. \tag{2}$$

If  $v_{A,t} = v_{B,t}$ , then player  $i$  randomizes with mixed probabilities over  $A$  and  $B$ ; that is,  $x_t^i = A$  with probability  $p$ , and  $x_t^i = B$  with probability  $1 - p$ , where  $p \in (0, 1)$ . This pair of actions determines the payoffs,  $\pi_t^1(x_t^1, x_t^2)$  and  $\pi_t^2(x_t^2, x_t^1)$

Player  $i$  updates valuations according to the sign of the difference between  $\pi$  and  $v$ . Whatever the size of the difference, the extent of revision for the valuation is constant. Let  $i \in \{1, 2\}, i \neq j$  and  $\alpha \in \{A, B\}$ ,

$$v_{\alpha,t+1}^i = \begin{cases} v_{\alpha,t}^i + 1 & \text{if } x_t^i = \alpha \text{ and } \pi_t^i(x_t^i, x_t^j) > v_{\alpha,t}^i \\ v_{\alpha,t}^i & \text{if } x_t^i = \alpha \text{ and } \pi_t^i(x_t^i, x_t^j) = v_{\alpha,t}^i \\ v_{\alpha,t}^i - 1 & \text{if } x_t^i = \alpha \text{ and } \pi_t^i(x_t^i, x_t^j) < v_{\alpha,t}^i. \end{cases} \tag{3}$$

If  $x_t^i \neq \alpha$ , then  $v_{\alpha,t+1}^i$  retains the same value. This updating rule expresses the fact that players change their valuations grid by grid; we assume the constant change of valuations.

These updating rules define a Markov process over the (social) state space defined by the set  $[L, H] \times [L, H] \times [L, H] \times [L, H]$ . Let  $S$  be the state space. The process will be denoted  $P$  and be referred to as the *untrembled process*.

Under the untrembled process there are in general many stationary states. If we introduce trembles in choosing actions, we can select the most robust outcome against

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<sup>1</sup> If necessary, we can transform payoffs and initial valuations by adding large positive numbers to them.

small perturbations. Let  $X^*$  be the action chosen under the untrembled process, and let  $x^1$  be another action. We assume that each player chooses  $X^*$  with probability  $1 - \varepsilon$  and  $x^1$  with  $\varepsilon$ , where  $\varepsilon$  is a small positive number less than 1. The process will be denoted  $P^\varepsilon$  and is referred to as the *trembled process*.

### III. Results

We first define the stable states in the untrembled process.

A stable state is such that, for all  $i, j \in \{1, 2\}$  and  $i \neq j$ , (1) it induces a pair of actions  $(x^1, x^2)$ ; (2) the valuation of the action is exactly equal to the achieved payoff, so that  $v_{xi,t}^i = \pi^i(x^i, x^j)$ ; and (3) the valuation of the action exceeds that of another one:  $v_{xi,t}^i > v_{yi,t}^i$  where  $x^i \neq j^i$ . We refer to the state as the pure strategy state (PSS), following (Karandikar et al. 1998). Of course, every PSS is a recurrent class of the untrembled process by definition.

For convergence to PSS, we have the following result. To keep the proof simple, suppose that for any  $i \in \{1, 2\}$ ,  $d \leq v_{A,0}^i \leq a$  and  $c \leq v_{B,0}^i \leq b$  or  $b \leq v_{B,0}^i \leq c$ . (“PSSs” will be the plural form of “PSS.”)

Proposition 1

Assume that for any  $i \in \{1, 2\}$ ,  $d \leq v_{A,0}^i \leq a$  and  $c \leq v_{B,0}^i \leq b$  or  $b \leq v_{B,0}^i \leq c$ .<sup>1</sup> The untrembled process converges to particular PSSs. PSSs are constituted only of Pareto efficient outcomes and risk dominant outcomes.

From the construction of the trembled process,  $P^\varepsilon$  is irreducible and aperiodic for every  $\varepsilon > 0$ . Hence, by standard results for Markov processes, it has a unique stationary distribution. Denote this by  $\mu^\varepsilon$ . Any initial distribution converges to it, so that for any initial distribution  $d$ ,  $dP^\varepsilon \rightarrow \mu$  as  $t \rightarrow \infty$ . According to standard stochastic evolutionary game analysis (Young 1998, Young 1993, Kandori et al. 1993),  $\lim_{\varepsilon \rightarrow 0} \mu^\varepsilon = \mu^*$  exists.

If  $\mu^*(s) > 0$ , the state  $s$  is called stochastically stable, where  $\mu^*(s)$  is the probability of state  $s$  given  $\mu^*$ . For the probability of the set of states, let  $\mu^\varepsilon(A) \equiv \sum_{s \in A} \mu^\varepsilon(s)$ .

We derive the following result relating to stochastically stable states:

Proposition 2

Assume that for any  $i \in \{1, 2\}$ ,  $d \leq v_{A,0}^i \leq a$  and  $c \leq v_{B,0}^i \leq b$  or  $b \leq v_{B,0}^i \leq c$ ,

The risk dominant outcome corresponds to stochastically stable states. Formally, let  $s^R$  is

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1 This assumption is not used in Theorem 1. But it is reasonable if there are some test periods for deciding valuations based on the realized payoff.

the set of states such that  $v_B^1 = v_B^2 = b$ ,  $v_B^1 > v_A^1$  and  $v_B^2 > v_A^2$ .

Then  $\lim_{\varepsilon \rightarrow 0} \mu^\varepsilon(s^R) = 1$ .

This assumption is not used in Theorem 1. But it is reasonable if there are some test periods for deciding valuations based on the realized payoff.

Next, assume that for  $i \in \{1, 2\}$ ,  $v_{A,0}^i, v_{B,0}^i \in [L, H]$ . Under this condition, for any  $\varepsilon > 0$ ,  $P^\varepsilon$  is not irreducible. The standard technique cannot therefore be applied in this case. However, the assertions of Proposition 2 do not change, so that we derive the following theorem:

Theorem 1 Assume that for any  $i \in \{1, 2\}$ ,  $v_{A,0}^i, v_{B,0}^i \in [L, H]$ .

(1) The risk dominant outcome corresponds to stochastically stable states in the model.

Formally, let  $s^R$  is the set of states such that  $v_B^1 = v_B^2 = b$ ,  $v_B^1 > v_A^1$  and  $v_B^2 > v_A^2$ , then  $\lim_{\varepsilon \rightarrow 0} \mu^\varepsilon(s^R) = 1$

(2) Any initial state converges to a stochastically stable state.

#### IV. Informal Sketches of Proofs

All proofs are straightforward but long, and are therefore set out in full in the Appendix. In this section we describe the intuitive basis of the proofs. For Proposition 1, if a sequence of states does not enter in the PSS, the sequence circulates. However, from the way of updating valuations and the tie-breaking rule, it is impossible. In the proof we show that, from arbitrary initial conditions, there is a path to PSS with positive probability.

For Proposition 2, the range of initial states is restricted. Hence it is easy to derive the minimum cost tree. The costs from (A, A)-type PSS to (B, B) can be shown to be less than the costs from (B, B)-PSS to (A, A), by comparing a cost tree whose root is in (B, B)-PSS and the minimum cost tree whose root is in (A, A)-PSS.

For Theorem 1, the range of initial states is extended. However, there is a unique recurrent class that is exactly the same as that of Proposition 2. In other words, if a state is PSS but is not included in the set of PSS in Proposition 2, then it is transient.

According to standard results for Markov chains, this implies that there exists a unique limit distribution of  $P^\varepsilon$  and, for any transient state, as  $t \rightarrow \infty$ , then its limit distribution of  $P^\varepsilon$  is 0. From these facts it is easy to prove the statement of Proposition 2 under this assumption, and that there is good convergence from any initial state.

#### V. Concluding Remarks

This paper has examined stochastically stable states in the coordination game that show a tension between Pareto efficiency and risk dominance. In the present model, each

agent has a valuation of the possible actions, which are updated according to a sign of the difference between the current valuation and the realized payoff. The agent chooses the action having higher valuation. In this situation, the risk dominant outcome is selected.

One of implications from our results is as follows: Assume that (1) in “Post-Capitalism”, information industry and soft economy is dominant, (2) in these industry and economy, the coordination among producers is important, and (3) dynamics within a firm is derived from the rule of thumb defined in this paper. Then, in “Post-Capitalism”, if someone designs the production scheme, he/she should not try to improve outcome that is Pareto efficient. The Risk-dominant outcome appears mostly in the long run. Hence, he/she have to make improve outcome that is risk dominant.

Let us now consider extensions to the analysis. First, the present results and proofs suppose that the domain of valuations is a positive integer, and that the extent of revision for the valuation is constant. It is therefore important to consider the case in which the extent of revision depends on the difference between the valuation and the realized payoff. Second, one might assume that the probability of trembling “depends on the state; for example, it could depend on the difference between a valuation and the realized payoff. Third, there is a generalization to a finite-population model. Finally, we should consider the appropriateness of the present model in light of the data from experimental analysis.

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